Lec 09: Cryptography (1)

CSED415: Computer Security

Spring 2025

Seulbae Kim

POSTECH

POHANG UNIVERSITY OF SCIENCE AND TECHNOLOGY

Administrivia

- Lab 02 deadline is approaching
  - Due: Friday, March 21
  - Attend office hours for help!

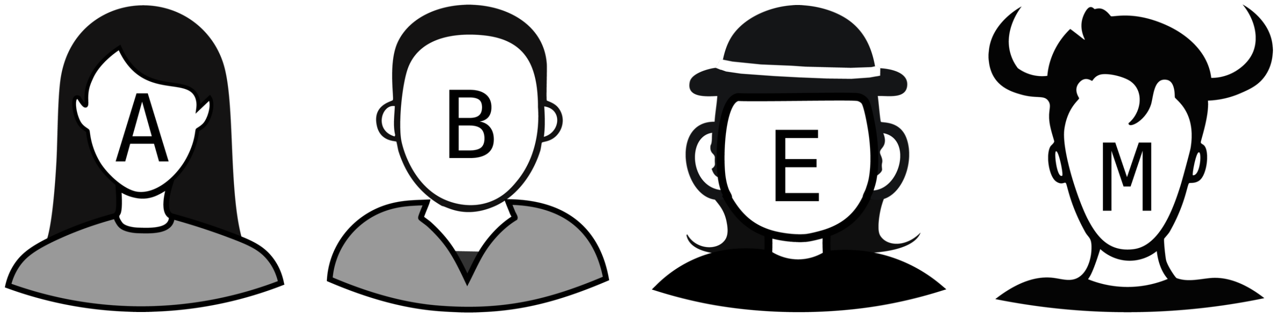
Cryptography – Definitions and Setting

What is cryptography?

- Definition:
  - A means to enable parties to maintain privacy of the information they send to each other, even in the presence of an adversary with access to the communication channel
- Cryptography enables secure communication over insecure channels

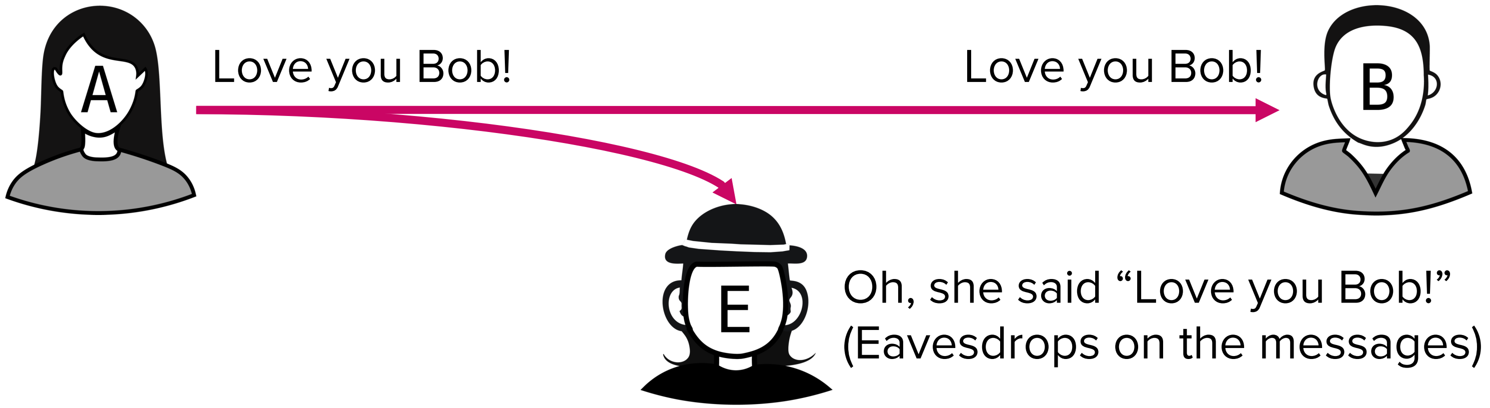
Main characters

- **Alice** and **Bob**: Two people who want to exchange messages over an insecure communication channel
- **Eve**: An eavesdropper who can read any data on the channel
- **Mallory**: A malicious adversary who can read and also modify any data on the channel

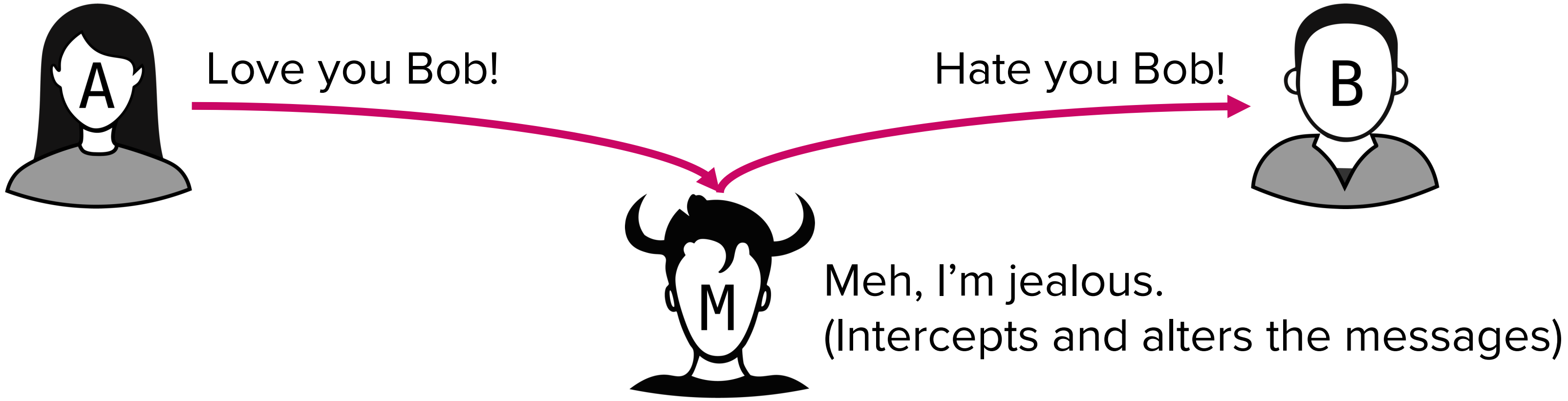


Cryptographic scenarios

- **Alice** & **Bob** against **Eve**



- **Alice** & **Bob** against **Mallory**



Goal: Preserving CI + A

- Three primary objectives of cryptography
  - **Confidentiality**: Ensuring that only authorized parties can access the contents of messages
  - **Integrity**: Guaranteeing that messages remain unaltered during transmission
  - **Authenticity**: Confirming the sender’s identity to verify that the message truly comes from the claimed source

Keys: The key to cryptography

- Keys control both the encryption and decryption
- Two key models:
  - Symmetric key model
    - Alice and Bob share the same key
  - Asymmetric key model
    - Each user has a secret key and a public key
      - Public key is shared to anyone
      - Secret key is kept confidential

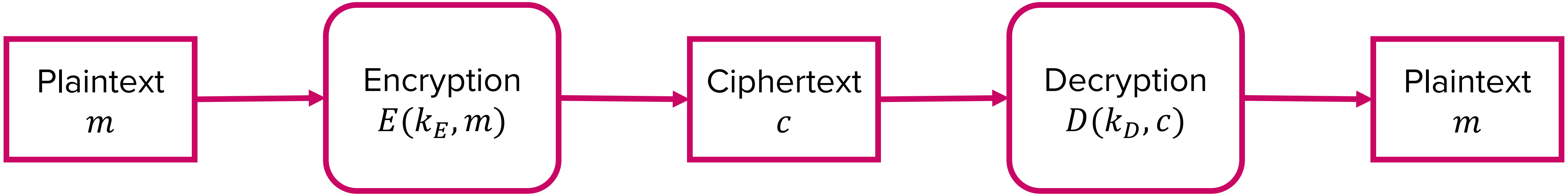
Kerckhoff’s principle

- “The security of a cryptosystem should not rely on the secrecy of its mechanism”
  - Cryptosystem should remain secure even when an attacker knows all internal details of the algorithm
  - The key should be the only thing that must be kept secret
  - Encourages the “Open Design” principle (ref: *Lecture 02*)
    - Security through obscurity is discouraged

We assume that an attacker knows everything except the secret key

Terms and notations

- Plaintext m: Original message
- Ciphertext c: Encrypted message
- Keys: An encryption key (k_E) and decryption key (k_D)
- Encryption $E(k_E, m)$: Process of generating c from m
- Decryption $D(k_D, c)$: Process of generating m from c



Cryptography roadmap

Goal \ Scheme	Symmetric Key	Asymmetric Key
Confidentiality	• One Time Pad (OTP) • Block ciphers (DES, AES) • Stream ciphers	• ElGamal encryption • RSA encryption
Integrity & Authentication	• Message Authentication Code (MAC)	• Digital signature

Classical Ciphers

Caesar cipher (58 BC)

- A basic substitution cipher:
  - Replaces each symbol with another symbol
- Algorithm
  - Key k: An integer within the range [0:25]
  - $E(k, m)$: Substitutes each letter in m with the letter that is k positions forward in the alphabet
  - $D(k, c)$: Substitutes each letter in c with the letter that is k positions backward in the alphabet

Caesar cipher

- Example
  - $k = 3$
  - m = HELLO WORLD
  - $E(k, m)$
    - H → K
    - E → H
    - L → O
    - …
  - c becomes KHOOR ZRUOG

Substitution table

m	c	m	c
A	D	N	Q
B	E	O	R
C	F	P	S
D	G	Q	T
E	H	R	U
F	I	S	V
G	J	T	W
H	K	U	X
I	L	V	Y
J	M	W	Z
K	N	X	A
L	O	Y	B
M	P	Z	C

Cryptanalysis of Caesar cipher

- Setting
  - Eve can see c = ORYH BRX ERE
  - Eve doesn't know k

- Possible attacks (1)
  - Brute-force attack: Try decrypting with all 26 possible keys

```
k=0  m=ORYH BRX ERE     k=8  m=GJQZ TJP WJW     k=16 m=YBIR LBH OBO     k=24 m=QTAJ DTZ GTG
k=1  m=NQXG AQW DQD     k=9  m=FIPY SIO VIV     k=17 m=XAHQ KAG NAN     k=25 m=PSZI CSY FSF
k=2  m=MPWF ZPV CPC     k=10 m=EHOX RHN UHU     k=18 m=WZGP JZF MZM
k=3  m=LOVE YOU BOB     k=11 m=DGNW QGM TGT     k=19 m=VYFO IYE LYL
k=4  m=KNUD XNT ANA     k=12 m=CFMV PFL SFS     k=20 m=UXEN HXD KXK
k=5  m=JMTC WMS ZMZ     k=13 m=BELU OEK RER     k=21 m=TWDM GWC JWJ
k=6  m=ILSB VLR YLY     k=14 m=ADKT NDJ QDQ     k=22 m=SVCL FVB IVI
k=7  m=HKRA UKQ XKX     k=15 m=ZCJS MCI PCP     k=23 m=RUBK EUA HUH
```

Cryptanalysis of Caesar cipher

- Setting
  - Eve can see c = ORYH BRX ERE
  - Eve doesn't know k

- Possible attacks (2)
  - Chosen-plaintext attack: Eve can choose arbitrary plaintexts and obtain their corresponding ciphertexts
    - e.g., by tricking Alice into encrypting m that Eve chose
  - Eve chooses m = ABCD and receives c = DEFG
    - Eve can readily deduce $k = 3$

Rail Fence cipher

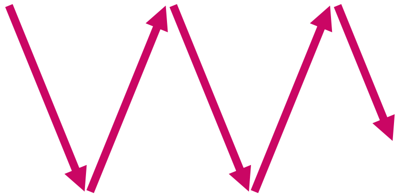
- A simple permutation cipher
  - Permutation cipher encrypts m by rearranging the letter order, without altering the actual letters used
- Scheme
  - Key k: An integer smaller than the length of plaintext m
  - $E(k, m)$:
    - Write the first letter of the plaintext
    - Write the following letters downwards diagonally for $k - 1$ letters, then write upwards diagonally for $k - 1$ letters
    - Repeat until the whole plaintext is written out

Rail Fence cipher

- Example
  - $k = 3$ (3 rails)
  - m = HELLO WORLD
  - $E(k, m)$:

```
H...O...L.
.E.L.W.R.D
..L...O...
```

→ c becomes `HOL ELWRD LO`



Cryptanalysis of Rail Fence cipher

- Vulnerable to brute-force attacks
  - k is always smaller than the length of m
  - An attacker can try decrypting c with all possible k's
- Vulnerable to exhaustive permutations (i.e., rearrangements)
  - c is a permutation of m
    - i.e., c is obtained by reordering m
  - Therefore, m is a permutation of c
  - An attacker can try all permutations of c to obtain m

Classical ciphers are considered weak

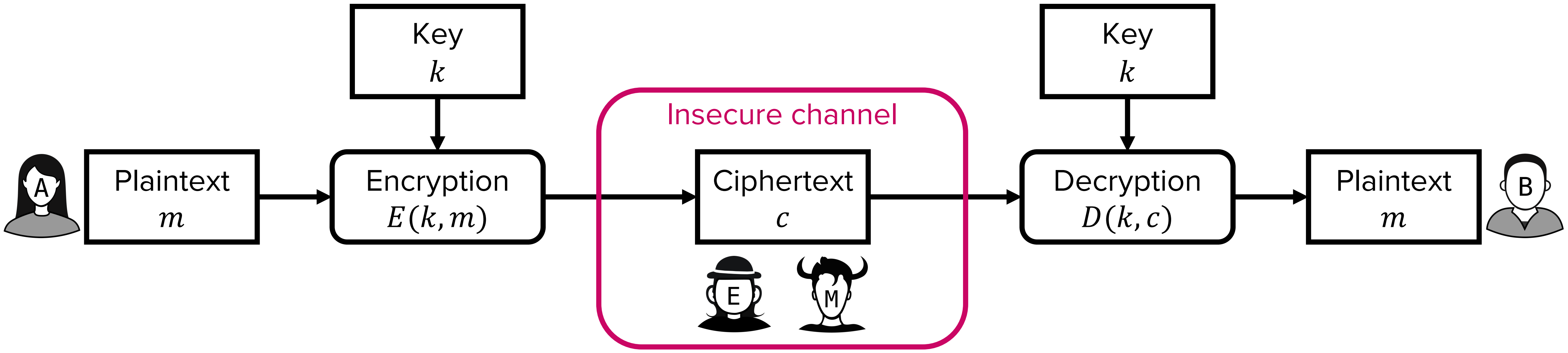
- Basic substitution cipher (S) and permutation cipher (P) are considered insecure
  - Reasons:
    - Letters in a natural language (e.g., English) are not uniformly distributed
    - Prior knowledge of letter frequencies (e.g., most frequent: e) can be used for cryptanalysis against S or P ciphers

What if we combine S with P?
→ Transition into modern cryptography

Symmetric Cryptography (Shared key Scheme)

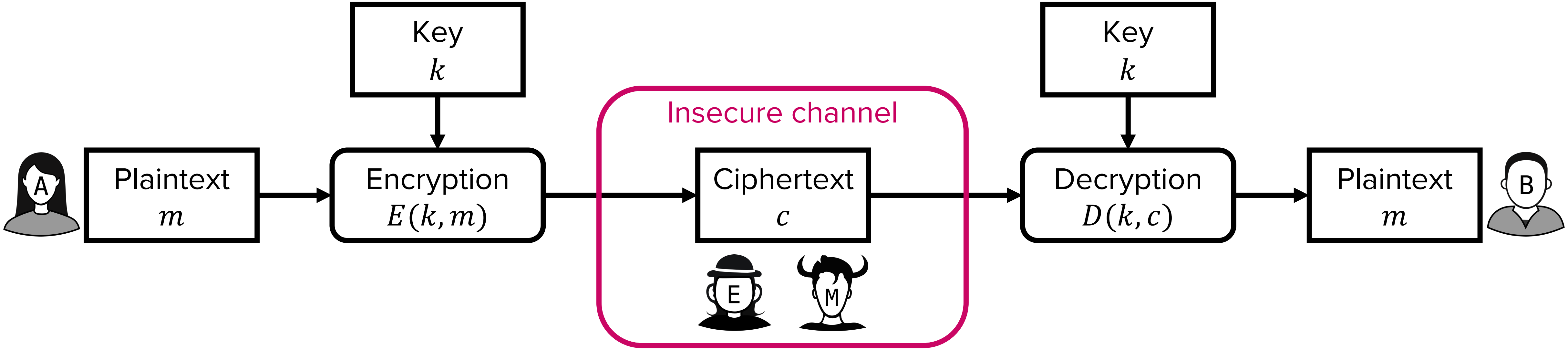
Symmetric key cryptography

- A symmetric encryption scheme consists of:
  - The key generation algorithm: Generates $k = k_\mathrm{E} = k_\mathrm{D}$ (symmetric!)
  - The encryption algorithm: $c = E(k, m)$
  - The decryption algorithm: $m = D(k, c)$



Symmetric key cryptography

- Required properties
  - Correctness
    - $D(k, E(k, m)) = m$ should hold for all k and m
  - Confidentiality
    - c should not give an attacker any additional information about m



One-time Pad (OTP)

- Scheme
  - Key k: Randomly selected bitstring of length n
    - n: length of the plaintext m
  - $E(k, m) = k \oplus m$: Bitwise XOR k and m
  - $D(k, c) = k \oplus c$: Bitwise XOR k and c

Review: XOR ($\oplus$)

$$0 \oplus 0 = 0 \qquad x \oplus 0 = x$$
$$0 \oplus 1 = 1 \qquad x \oplus x = 0$$
$$1 \oplus 0 = 1 \qquad x \oplus y = y \oplus x$$
$$1 \oplus 1 = 0 \qquad (x \oplus y) \oplus x = y$$

One-time Pad (OTP)

- Example
  - m = `OMW` (== bitstring $01001111\ 01001101\ 01011001$)
    - $n = 24$
  - $k = 00111101\ 01101010\ 11001101$
    - Generated at random, shared between Alice and Bob

One-time Pad (OTP)

- Example
  - Encryption (Alice)

m	0	1	0	0	1	1	1	1	0	1	0	0	1	1	0	1	0	1	0	1	1	0	0	1
	⊕	⊕	⊕	⊕	⊕	⊕	⊕	⊕	⊕	⊕	⊕	⊕	⊕	⊕	⊕	⊕	⊕	⊕	⊕	⊕	⊕	⊕	⊕	⊕
k	0	0	1	1	1	1	0	1	0	1	1	0	1	0	1	0	1	1	0	0	1	1	0	1
	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
c	0	1	1	1	0	0	1	0	0	0	1	0	0	1	1	1	1	0	0	1	0	1	0	0

  - Alice transmits c to Bob

One-time Pad (OTP)

- Example
  - Decryption (Bob)

c	0	1	1	1	0	0	1	0	0	0	1	0	0	1	1	1	1	0	0	1	0	1	0	0
	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$	$\oplus$
k	0	0	1	1	1	1	0	1	0	1	1	0	1	0	1	0	1	1	0	0	1	1	0	1
	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
m	0	1	0	0	1	1	1	1	0	1	0	0	1	1	0	1	0	1	0	1	1	0	0	1

  - Bob retrieves m = 01001111 01001101 01011001 = `OMW`

One-time Pad (OTP)

- Evaluation: Correctness
  - Cryptographic algorithm is correct if $D\big(k, E(k, m)\big) = m$

$$E(k,m) = k \oplus m \quad \cdots \text{ Definition of } E$$

$$\begin{aligned} D\big(k, E(k,m)\big) &= D(k, k \oplus m) && \cdots \text{ Substitution} \\ &= k \oplus (k \oplus m) && \cdots \text{ Definition of } D \\ &= m && \cdots \text{ Property of XOR} \end{aligned}$$

Thus, OTP is correct. ■

How do we evaluate the security (i.e., confidentiality)?

Theorem: Shannon's perfect secrecy (1949)

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- An encryption scheme is perfectly secure if for every ciphertext c and messages $m1$ and $m2$,

$$Prob[E(\mathcal{K}, m_1) = c] = Prob[E(\mathcal{K}, m_2) = c]$$

  - $\mathcal{K}$ is a random variable that is uniformly distributed over the key space $k \in \{0,1\}^n$ (a bitstring of length n)
- In plain English, even if an attacker has infinite time and computational powers in the world, he or she cannot crack your ciphertext if your scheme is Shannon-secure

OTP ensures perfect secrecy

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- Theorem

$$\forall c, \forall m_1, \forall m_2$$

$$Prob[E(k, m_1) = c] = Prob[E(k, m_2) = c]$$

- Proof
  - Fix any ciphertext $c \in \{0,1\}^n$ (i.e., a bitstring of length n)
  - For every m, $Prob[E(k, m) = c] = Prob[k = m \oplus c] = 2^{-n}$
    - Constraint: For every new message m, a new key k is generated

OTP ensures perfect secrecy

- Example
  - m = `SEE YOU AT 8PM TOMORROW`
  - $c = 001010001 \ldots$
  - Attacker tries **all** possible $k \in \{0,1\}^n$ and decrypt the given c
    - What the attacker gets:

```
SEE YOU AT 2PM TOMORROW
EAT HIM BY 4PM TOMORROW
THE CAT IN THE HOSPITAL
WAS JIM AT THE VINEYARD
          ...
```

→ Can NEVER guess the correct m

Why not use OTP everywhere?

- Practical limitations exist
  - Key generation: Each k should be used only once
    - k needs to be randomly generated for each message (expensive)
  - Key management: k needs to be as long as m
    - Storage complexity increases for longer m
  - Key distribution: k needs to be shared
    - n-bit k needs to be shared securely first before we can send c securely

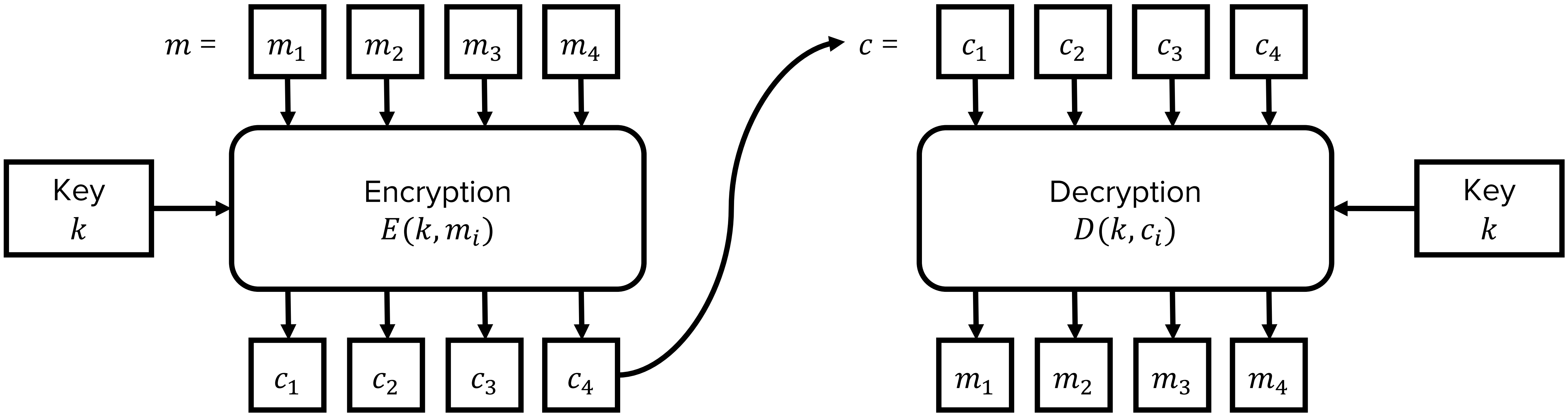
OTP is impractical for real-world usage

Cryptography roadmap

Goal \ Scheme	**Symmetric Key**	**Asymmetric Key**
Confidentiality	✅ One Time Pad (OTP) • Block ciphers (DES, AES) • Stream ciphers	• ElGamal encryption • RSA encryption
Integrity & Authentication	• Message Authentication Code (MAC)	• Digital signature

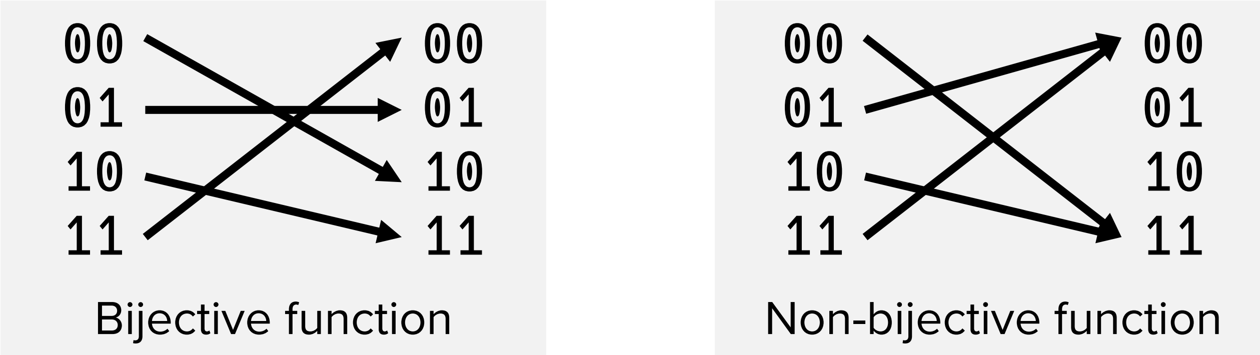
Block ciphers

- A scheme consisting of encode/decode algorithms for a fixed-sized block of bits



Correctness requirement of block ciphers

- E: A permutation (bijective function) and D: E^{-1} (inverse of E)
  - Every input is uniquely mapped to a single output



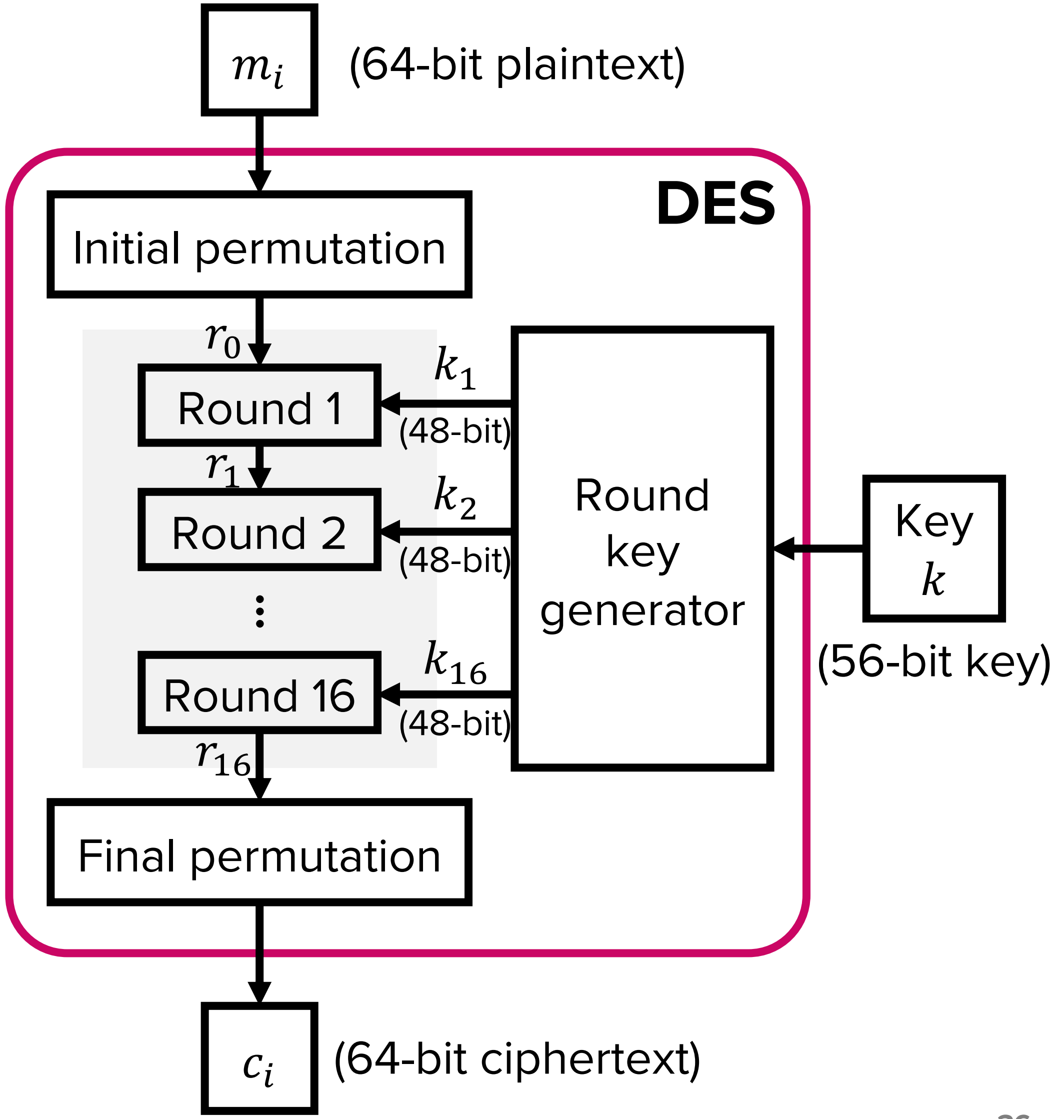
  - If E is not bijective, there may exist m_1 and m_2 such that

$$E(k, m1) = E(k, m2) = c$$

  - Then, we cannot decode c and obtain a unique plaintext

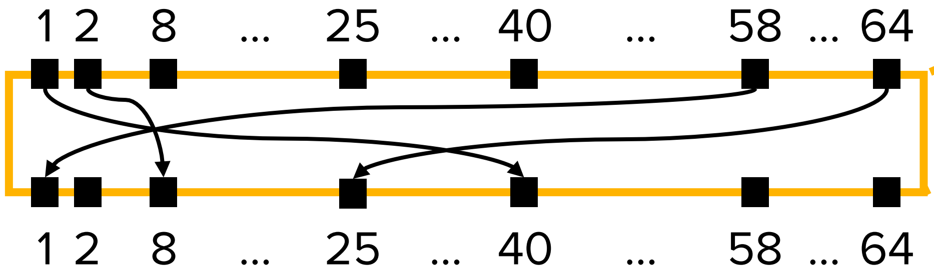
DES (Data Encryption Standard) (1975)

- Setting
  - Key size: 56 bits
  - Block size: 64 bits
    - In: 64-bit plaintext
    - Out: 64-bit ciphertext

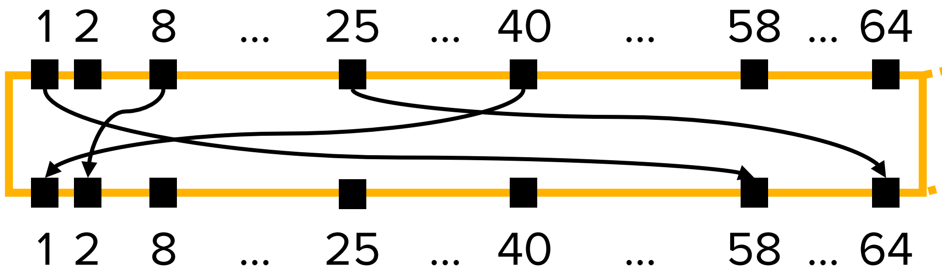


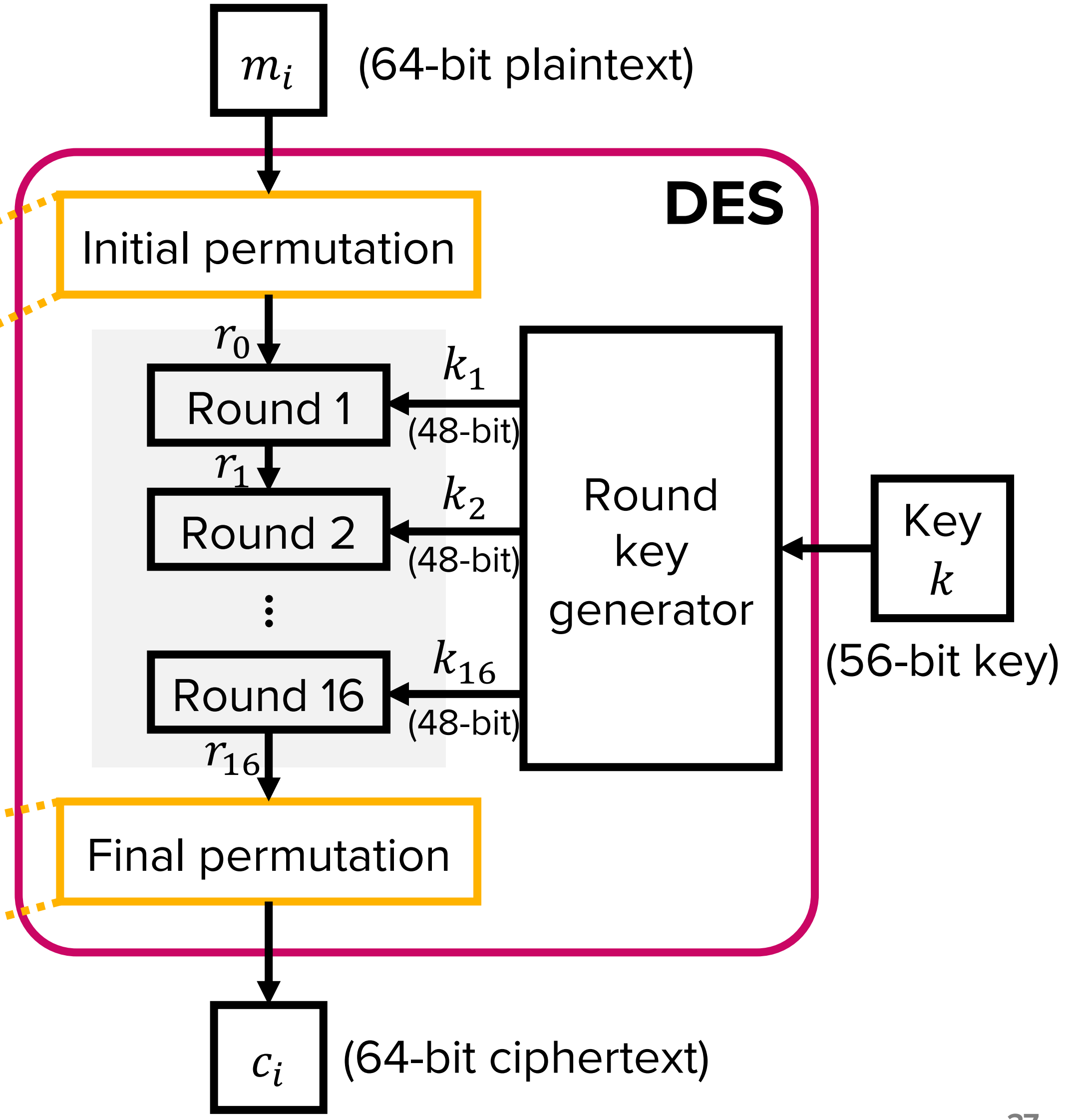
DES (Data Encryption Standard) (1975)

- Initial permutation (IP)
  - Rearranges the bits of m (diffusion)



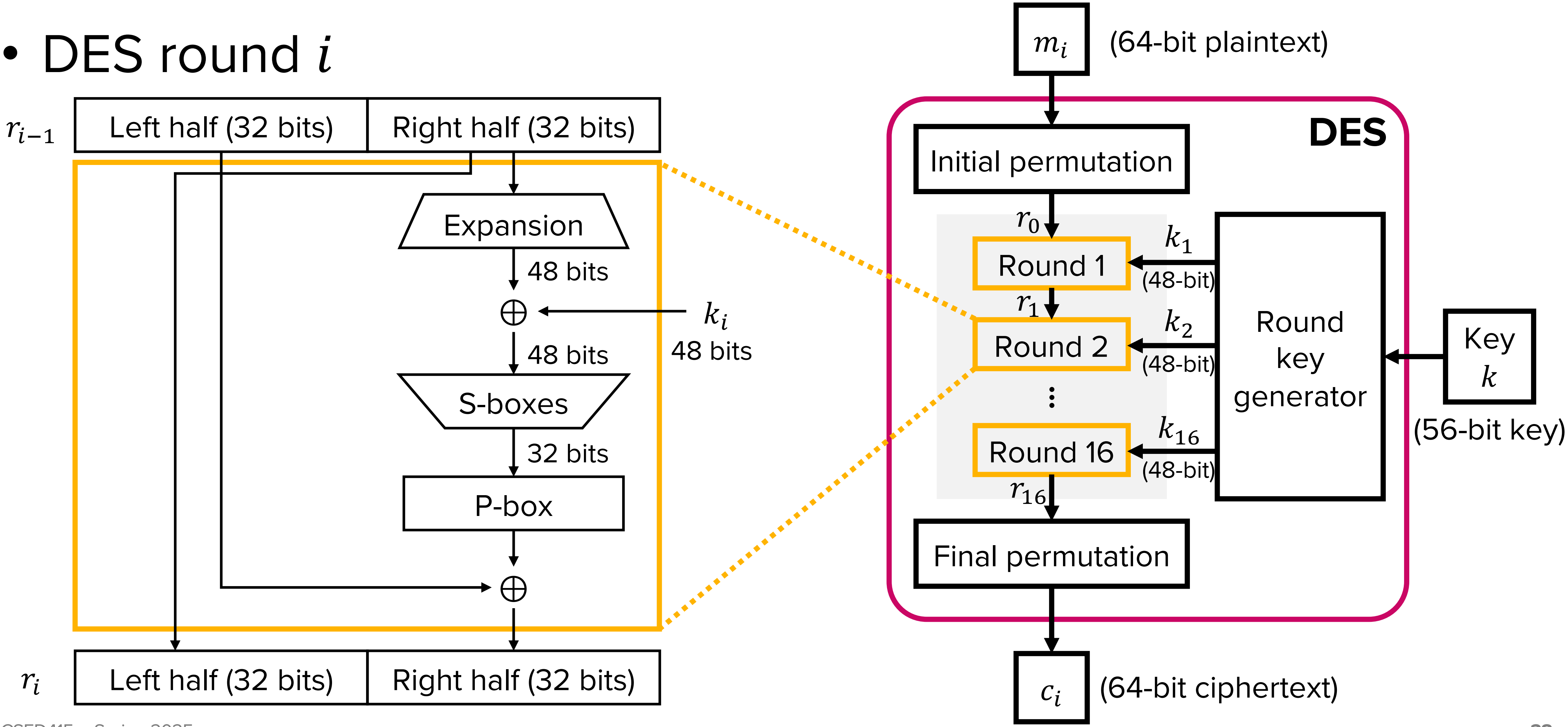
- Final permutation
  - Inverse of the IP





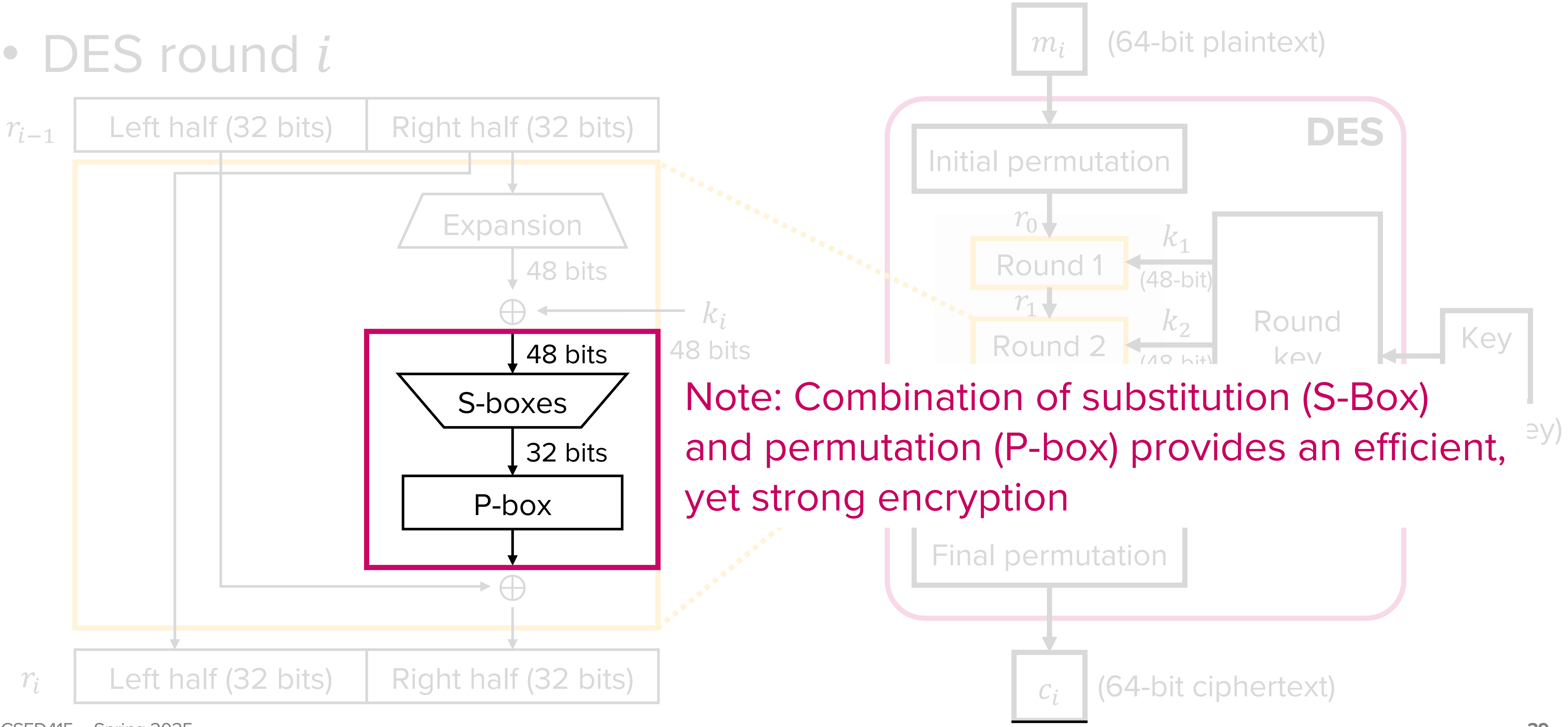
DES (Data Encryption Standard) (1975)

- DES round i



DES (Data Encryption Standard) (1975)

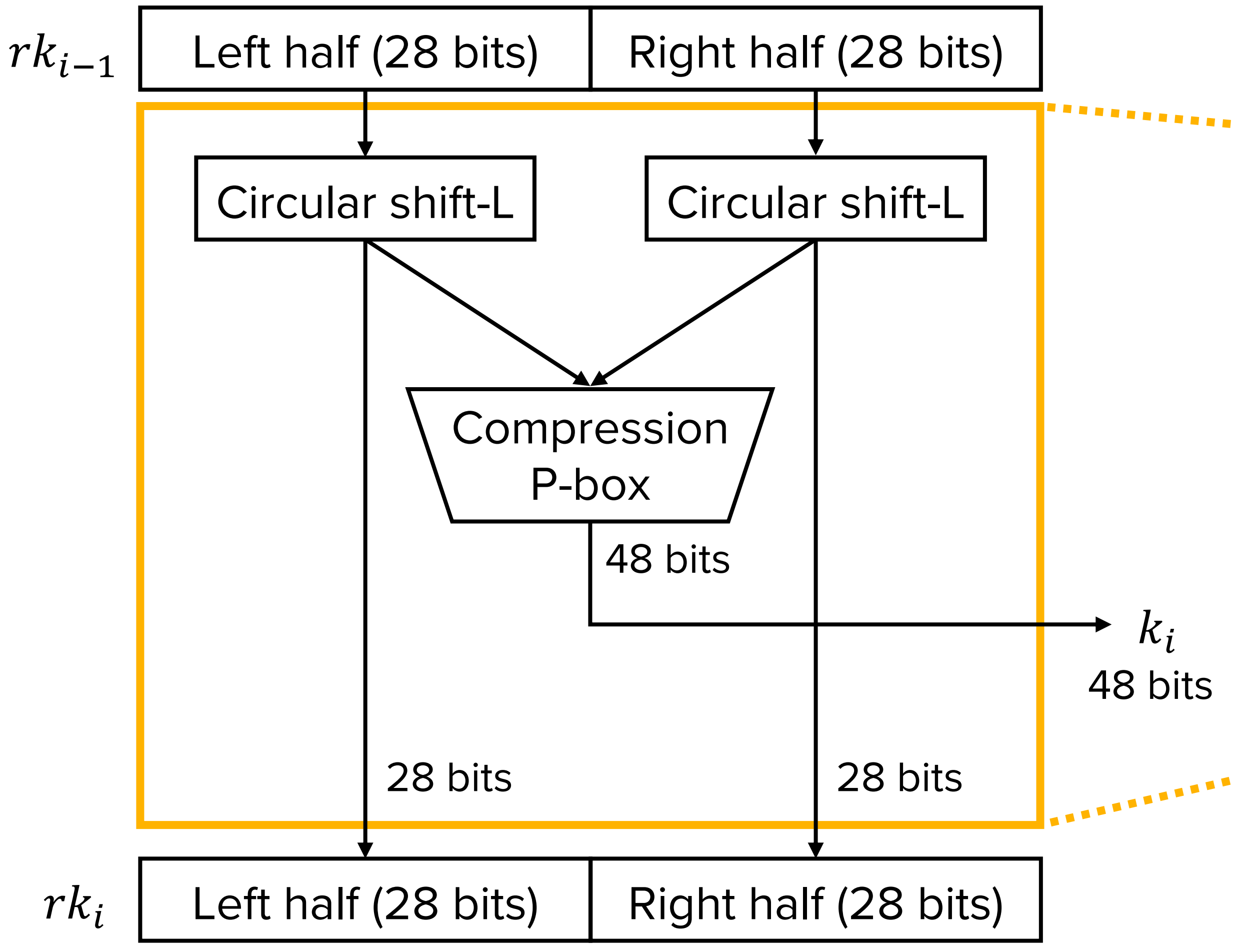
- DES round i

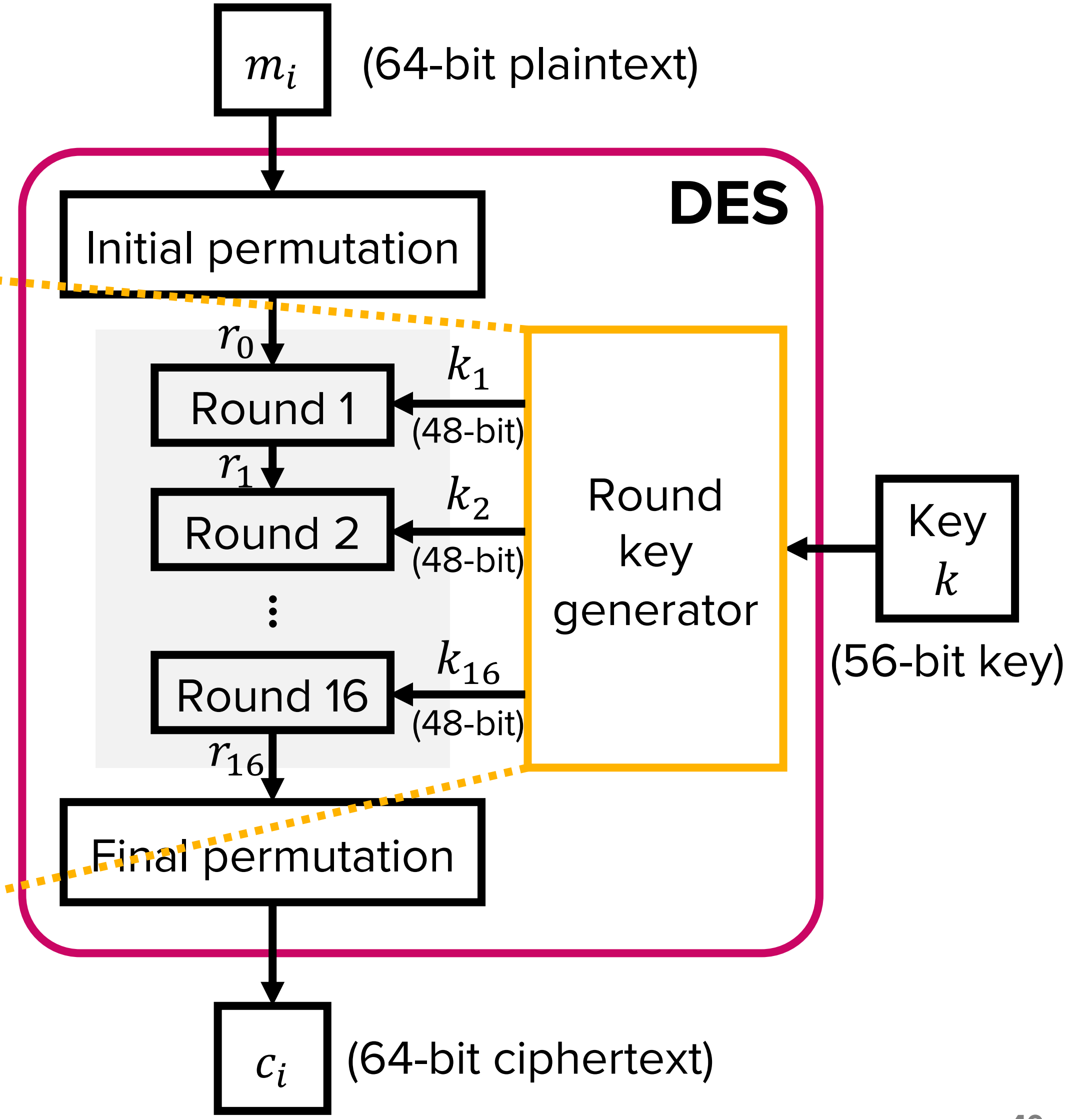


Note: Combination of substitution (S-Box) and permutation (P-box) provides an efficient, yet strong encryption

DES (Data Encryption Standard) (1975)

- DES round key (k_i) generation





Cryptanalysis of DES

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- DES algorithm itself remains unbroken even now
  - No algorithmic weakness has been identified yet
- However, DES is considered unsafe due to its small key size
  - The entire keyspace of a 56-bit key can be searched within days on modern computers
  - In 1999, a dedicated machine brute-forced DES key in 22 hours (ref: *Lecture 01*)
  - A replacement cipher was needed

Triple-DES (3DES)

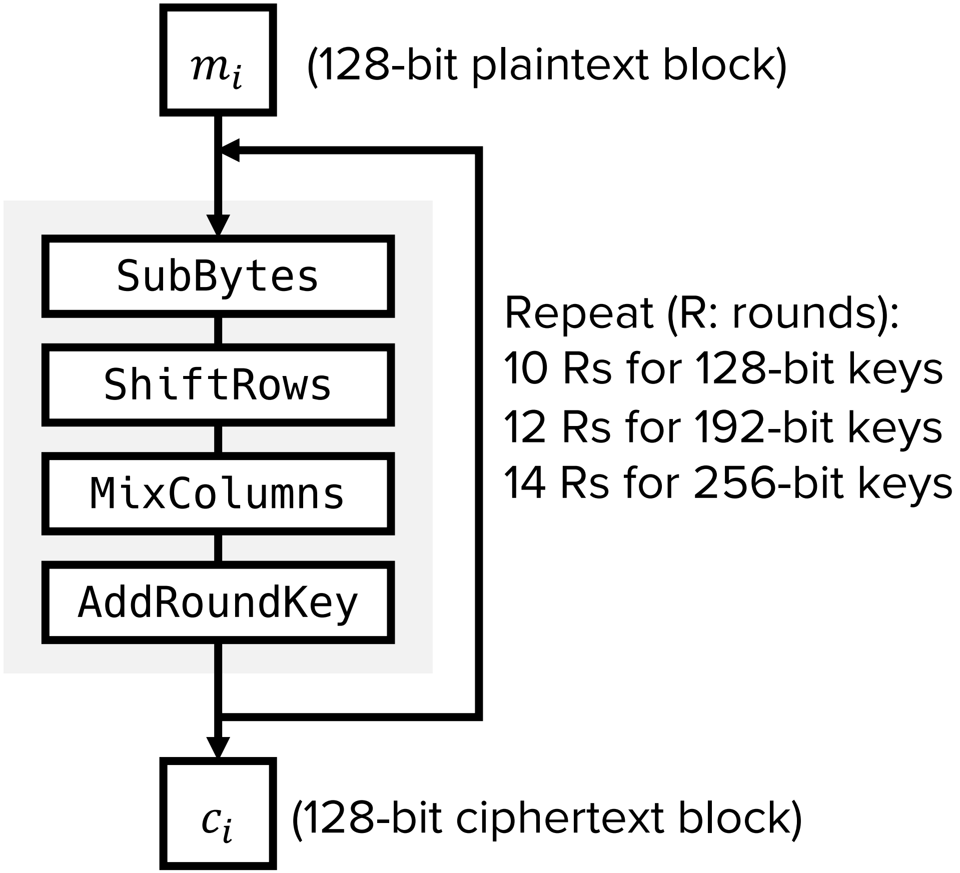
- Extends DES by applying DES three times
  - Use two keys: k_1 and k_2 (Key size is 56*2 = 112 bits)
  - $3DES(k_1, k_2, m) = DES\left(k_2, DES^{-1}\left(k_1, DES(k_2, m)\right)\right)$
  - Q) Why perform Enc-Dec-Enc, not Enc-Enc-Enc? Think about it!
- Cryptanalysis
  - Underlying encryption algorithm (DES) is the same
  - Security: Since key size is larger, brute-force attacks are much more challenging
  - Efficiency: Bad because 3DES requires three DES computations

AES (Advanced Encryption Standard) (2001)

- A new encryption standard replacing DES
  - 15 algorithms from different countries were submitted to NIST
  - Rijndael algorithm by John Daemen and Vincent Rijmen was selected as the Advanced Encryption Standard
- Setting
  - Key size: 128, 192, or 256 bits
  - Block size: 128 bits

AES (Advanced Encryption Standard) (2001)

- Scheme
  - m_i: A 128-bit block (4x4)
  - Repeat multiple rounds of:
    - `SubBytes`: Substitute bytes within block
    - `ShiftRows`: Shift bytes in each row
    - `MixColumns`: Multiply columns
    - `AddRoundKey`: XOR with round key



*You do not need to know all details of AES

Cryptanalysis of AES

- AES has not been broken
  - No algorithmic weakness
  - Exhaustive key search is believed to be infeasible
    - Nor formally proven, but empirically, no practical attack has been discovered
    - 128-bit key is large enough to prevent brute-force attacks

- Stronger and faster than DES/3DES

→ AES remains the de facto standard for block ciphers

Cryptography roadmap

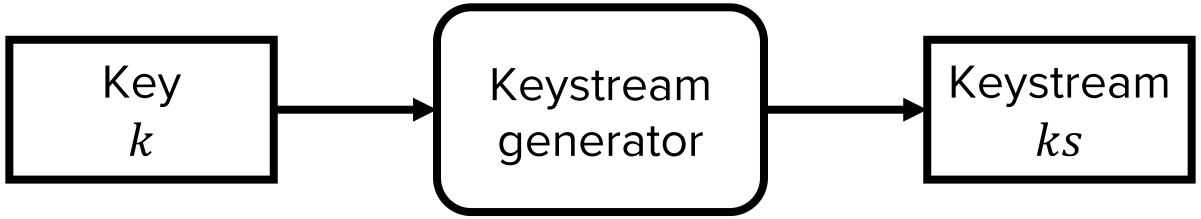
Scheme / Goal	**Symmetric Key**	**Asymmetric Key**
Confidentiality	✅ One Time Pad (OTP) ✅ Block ciphers (DES, AES) • Stream ciphers	• ElGamal encryption • RSA encryption
Integrity & Authentication	• Message Authentication Code (MAC)	• Digital signature

Stream Cipher

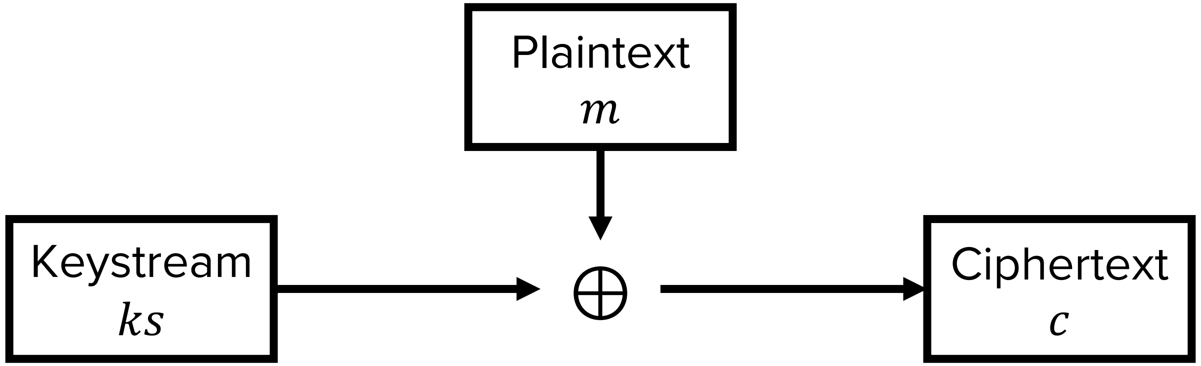
- Block ciphers split plaintext message into equal-sized blocks and encrypt each block as a unit
  - Overhead is introduced for block-granularity processing (e.g., need to add padding for messages smaller than the block size)
- Stream ciphers encrypt one bit at a time
  - Provide better efficiency in real-time communications

Stream cipher – Approach

- Generate a pseudorandom keystream ks from k



- $E(ks, m)$: Bitwise XOR keystream ks with plaintext m



Background: Randomness

- Randomness is essential for symmetric key cryptography
  - e.g., Stream cipher requires a random keystream
- If an attacker can predict a random number, many cryptographic schemes will be broken
- How can we securely generate random numbers?
  - Can computers generate random numbers?

Background: Randomness

- Entropy: A measure of uncertainty
  - High entropy means the outcomes are more unpredictable, which is desirable in cryptography
  - The uniform distribution has the highest entropy
    - e.g., Every output of a coin toss is equally likely

- In cryptography, randomness indicates uncertainty

Background: Randomness

- Keystream generator scenario
  - We want a keystream for stream cipher that attacker cannot guess
  - We can generate every bit of ks by tossing a fair (50-50) coin
  - Attacker cannot feasibly guess ks due to high entropy
    - “This ks is truly random”

  - Problem?

How would a computer do this?

Background: True randomness

- True randomness requires a physical source of entropy
  - A physical coin toss
  - Chaotic systems with complex dynamics, e.g., weather patterns
  - Atmospheric noise
  - Human activity

→ Very expensive and slow to generate

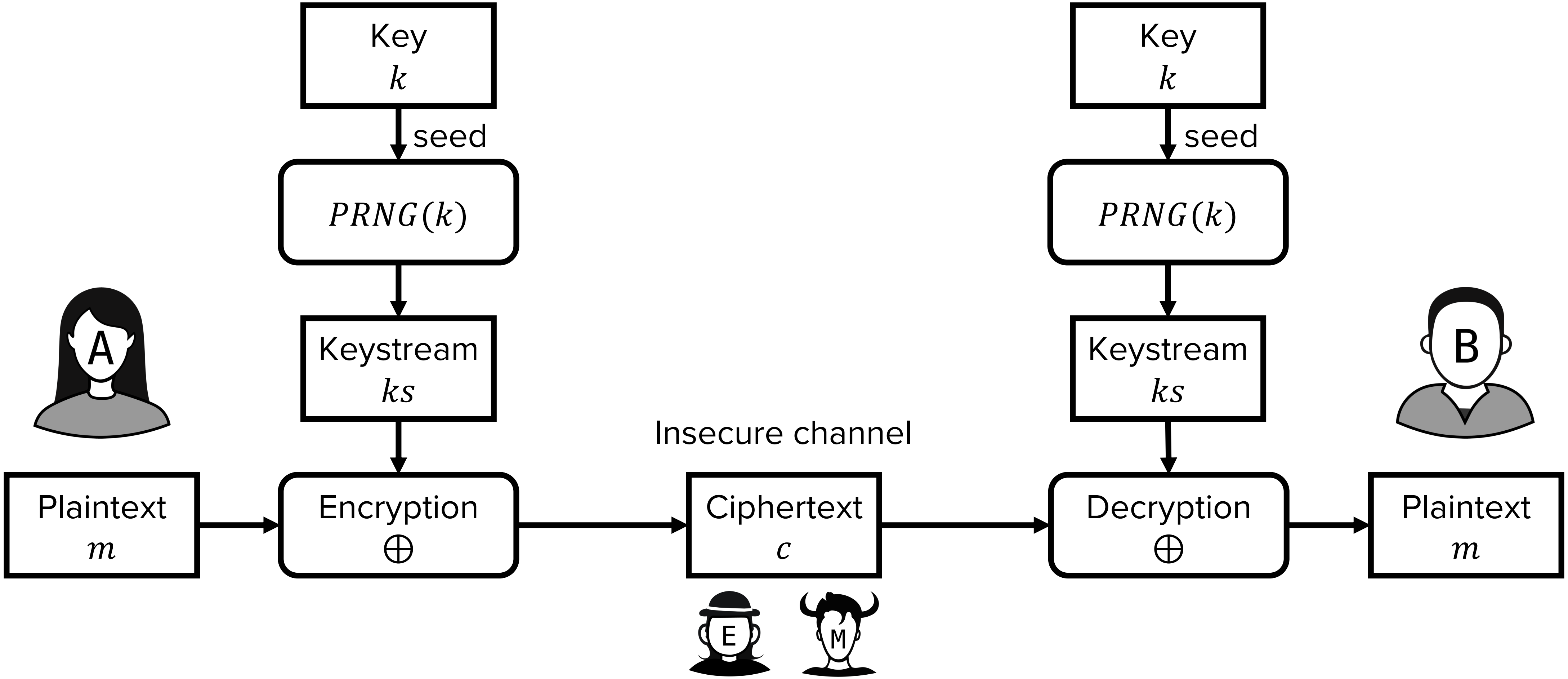
Again, how would a computer do this?

Background: Pseudo-Random Number Generator

- PRNG: An algorithm that utilizes a small seed of true randomness to produce outputs that appear random
- Procedure
  - Generate a seed from expensive true randomness
    - e.g., environmental noise from device drivers, such as keystroke intervals
  - Seed a PRNG algorithm
  - Generate pseudorandom numbers quickly and cheaply
- PRNG outputs are deterministic, yet computationally indistinguishable from true random numbers

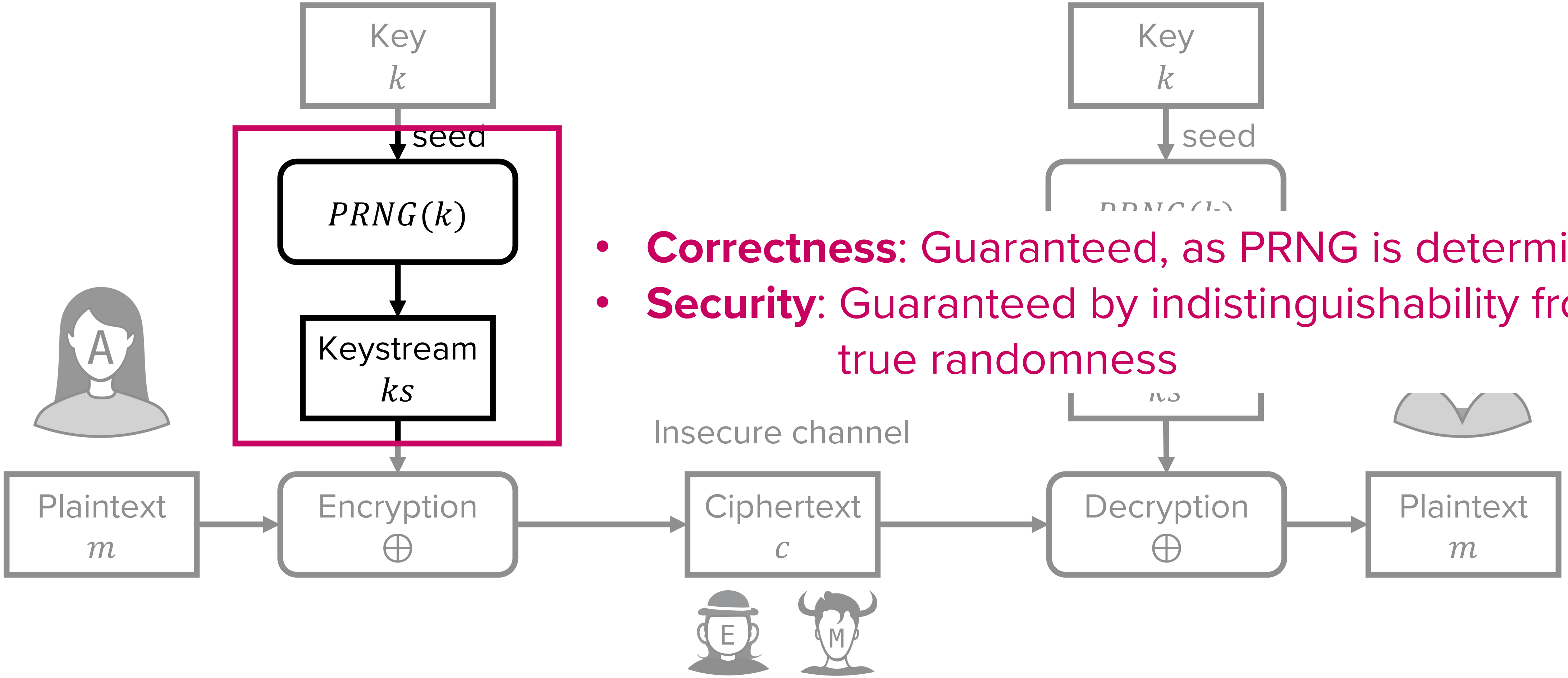
Back to stream cipher…

- Setting



Evaluating stream cipher

- Setting



- **Correctness**: Guaranteed, as PRNG is deterministic
- **Security**: Guaranteed by indistinguishability from true randomness

Example: Rivest Cipher (RC4) (1987)

- A classical stream cipher
  - Generates a continuous keystream ks of pseudorandom bytes from a secret key k
  - Encrypts plaintext m by XORing ks with m
- Variable-length key k: 5 to 256 bytes (let's assume 256 bytes)
  - Each byte of k can be accessed via $k[0]$, $k[1]$, …, $k[255]$, where $k[i]$ denotes the $i + 1$-th byte of k
- Consists of a Key Scheduling Algorithm (KSA) and Pseudo-Random Generation Algorithm (PRGA)

Example: Rivest Cipher (RC4) (1987)

- Key scheduling algorithm (KSA):
  - Initializes the S-Box array S
  - Given: Key k = $k[0]$, $k[1]$, …, $k[255]$
  - Initial S-box array: $S[0] = 0$, $S[1] = 1$, …, $S[255] = 255$

```python
def KSA(k):
  S = list(range(256)) # S = [0, 1, ..., 255]
  j = 0
  for i in range(256):
    j = (j + S[i] + k[i]) % 256 # %: modulo
    S[i], S[j] = S[j], S[i] # swap

  return S
```

Example: Rivest Cipher (RC4) (1987)

- Pseudo-Random Generation Algorithm (PRGA):
  - Generates a pseudorandom keystream ks
  - Given: S-box array = $S[0]$, $S[1]$, …, $S[255]$ (initialized by KSA)

```
def PRGA(m, S):
  i, j = 0
  ks = []
  for l in range(len(m)): # ks should be as large as plaintext
    i = (i + 1) % 256
    j = (j + S[i]) % 256
    S[i], S[j] = S[j], S[i] # swap
    t = (S[i] + S[j]) % 256
    ks[l] = S[t]
    l += 1
  return ks
```

Example: Rivest Cipher (RC4) (1987)

- Encryption:
  - Bitwise-XOR m with ks generated by PRGA
    - i.e., $c = m \oplus ks$
- Decryption
  - Generate ks from secret key k via KSA and PRGA
  - Bitwise-XOR c with ks
    - i.e., $m = c \oplus ks$

Example: Rivest Cipher (RC4) (1987)

- Security of RC4
  - Many known weaknesses exist
    - Key-dependent biases occur in the initial bytes of ks
    - Inferable correlation between keystream and the key
    - …
  - Despite its efficiency and simplicity, RC4 is no longer recommended for cryptographic applications
    - Secure alternatives: ChaCha20, AES-CTR, …

Cryptography roadmap

Scheme / Goal	Symmetric Key	Asymmetric Key
Confidentiality	✅ One Time Pad (OTP) ✅ Block ciphers (DES, AES) ✅ Stream ciphers	• ElGamal encryption • RSA encryption
Integrity & Authentication	• Message Authentication Code (MAC)	• Digital signature

Coming up next

- Limitations of symmetric schemes
  - Key needs to be securely shared
  - Too many keys are needed
    - One key for 2 ppl, 3 keys for 3 ppl, 6 keys for 4 ppl, 10 keys for 5 ppl, …

→ Asymmetric schemes were introduced

Questions?