# Lec 12: Digital Signatures and Certificates

## CSED415: Computer Security

Spring 2025

**Seulbae Kim**

POSTECH
POHANG UNIVERSITY OF SCIENCE AND TECHNOLOGY

# Administrivia

- Lab 03:
  - Due: Friday, April 4
- 10-minute proposal presentations:
  - When: Thursday, April 3, during regular class time
- Midterm exam:
  - When: Tuesday, April 8, during regular class time

# Cryptography roadmap

| Goal \ Scheme | Symmetric Key | Asymmetric Key |
| --- | --- | --- |
| **Confidentiality** | ✅ One Time Pad (OTP)<br>✅ Block ciphers (DES, AES)<br>✅ Stream ciphers | ✅ DH secure key exchange<br>✅ ElGamal encryption<br>✅ RSA encryption |
| **Integrity & Authentication** | ✅ Message Authentication Code (MAC) | • Digital signature |
| **CIA at the same time** | ✅ Authenticated encryption | |

**Additional Tool**

✅ Hash functions

# Digital Signatures

# Missing integrity and authenticity

- Asymmetric encryption (like symmetric encryption) only provides confidentiality, not integrity
  - Message Authentication Code (MAC) solves the integrity problem in the symmetric-key setting

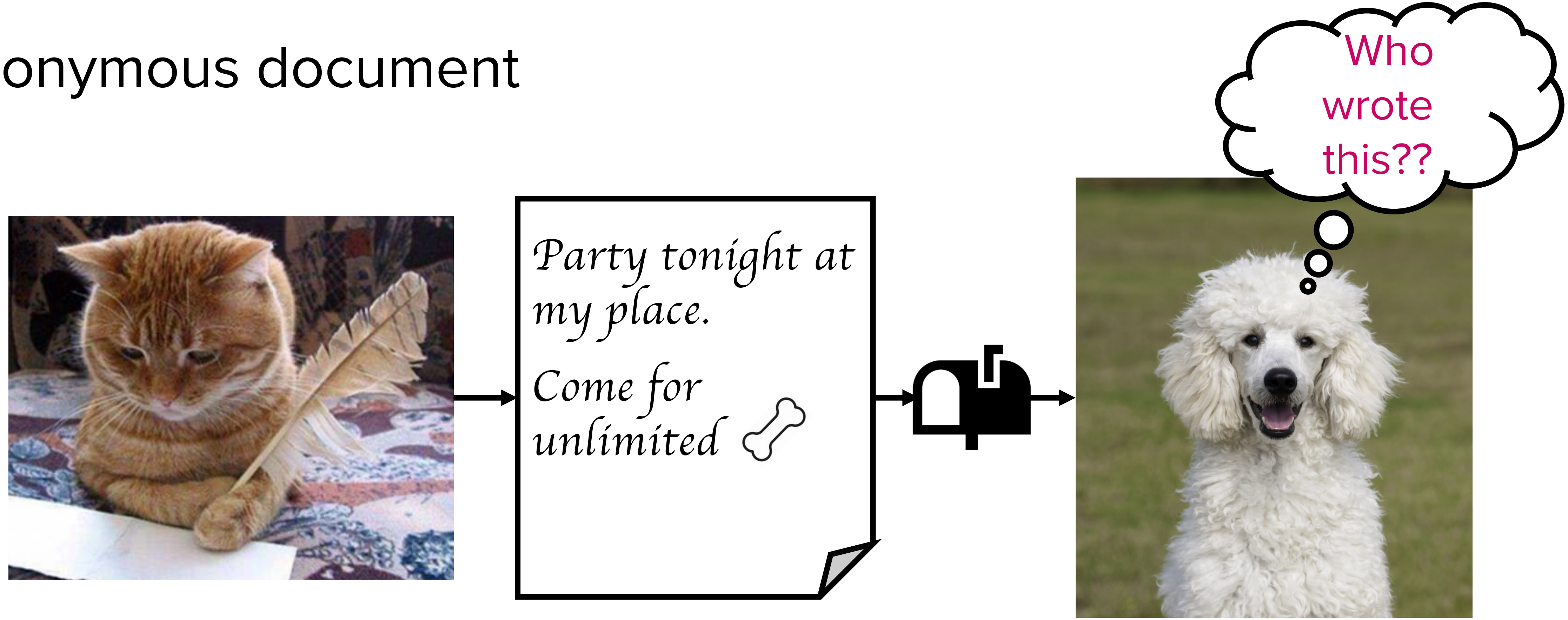
→ Question: Can we use asymmetric cryptography to provide integrity and authenticity of messages?

# Authenticity in real life

- Anonymous document

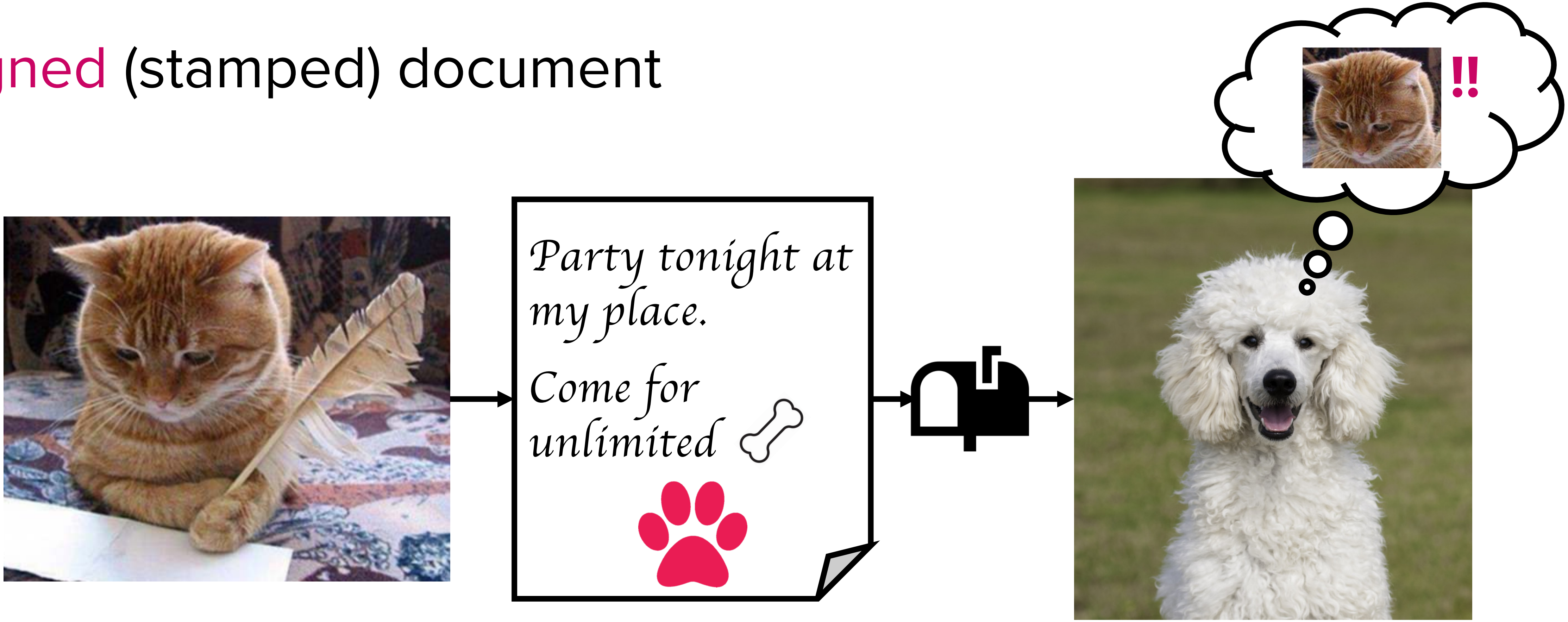
# Authenticity in real life

- Anonymous document



# Authenticity in real life

- Signed (stamped) document



# Digital signatures

- Key idea:
  - In asymmetric cryptography, each user has a secret key $k_s$ and a public key $k_p$
  - Only the **owner of the secret key** $k_s$ **can sign** a message with the secret key
  - **Everyone else can** verify the signature using the corresponding public key $k_p$

# Digital signatures

- Method:
  - Given: A key pair $< k_p, k_s >$
    - $k_s$: private key (also known as signing key or secret key)
    - $k_p$: public key
  - $S(k_s, m)$: Sign $m$ using secret key $k_s$ to generate signature $\sigma$
  - $V(k_p, m, \sigma)$: Verify signature $\sigma$ of message $m$ using public key $k_p$

# Difference in key usage

- Note: Digital signatures use key pair in the opposite order of asymmetric encryption schemes
  - Asymmetric encryption:
    - Alice (sender) encrypts using Bob's (receiver's) public key $k_p$
    - Bob (receiver) decrypts using his (receiver's) secret key $k_s$
  - Digital signature:
    - Alice (sender) signs using her (sender's) secret key $k_s$
    - Bob or anyone (receiver) verifies using Alice's (sender's) public key $k_p$

# MAC vs Digital signature

- When using MAC (symmetric cryptography):
  - The verifier must share a secret (key $k$) with the sender
  - Consequently, the verifier could potentially impersonate the sender!
    - e.g., by generating MAC tags using the shared key
- When using digital signatures (asymmetric cryptography):
  - The verifier utilizes the sender's public key
    - Does not require any shared secret
  - Consequently, the verifier cannot impersonate the sender!
    - Only who owns the private key (i.e., the sender) can generate valid signatures

# Security of DS

- DS scheme


- Metric to evaluate security (Same as MAC's)
  - **Unforgeability**: No polynomial time adversary should be able to produce forgery (i.e., $m$ and sig $\sigma$, where $m$ was never queried to $S$) with non-negligible probability, even after seeing multiple legitimate $< m, \sigma >$ pairs

# Naïve DS: Sign $m$

- Let's utilize the Vanilla RSA encryption for building $S$ and $V$
  - Recall RSA:
    - Select two large primes $p$ and $q$. $N = pq$
    - Compute totient $T = (p-1)(q-1)$
    - Select $k_p$, which is coprime to $T$ // $k_p = e$ (notation we used in Lecture 10)
    - Compute $k_s = k_p^{-1} \; mod \; T$ // $k_s = d$ (notation we used in Lecture 10)
    - Ciphertext $c \leftarrow E(k_p, m) = m^{k_p} \; mod \; N$
    - Decrypted $m \leftarrow D(k_s, c) = c^{k_s} \; mod \; N$

→ Key property (Euler's theorem): $m^{k_p k_s} \; mod \; N = m$

Here, the order of $k_p$ and $k_s$ does not matter!

# Naïve DS: Sign $m$

- Let's utilize the Vanilla RSA encryption for building $S$ and $V$
  - Given: Message $m$, secret key $k_s$, public $k_p$, public parameter $N$
  - Alice signs $m$ via $S(k_s, m)$: $\sigma \leftarrow m^{k_s} \ mod \ N$
  - Send $m$ and $\sigma$ to Bob
  - Bob verifies the signature $\sigma$ via $V(k_p, \sigma, m)$:
    - $m' \leftarrow \sigma^{k_p} \ mod \ N$ // message retrieved by decrypting $\sigma$
    - If $m = m'$ then return 1, else return 0

→ Can an attacker forge a valid pair $< m, \sigma >$?

Yes! Any attacker can forge $m = 1$ and $\sigma = 1$.

Verification: $m' \leftarrow \sigma^{k_p} \ mod \ N = 1^{k_p} \ mod \ N = 1.$

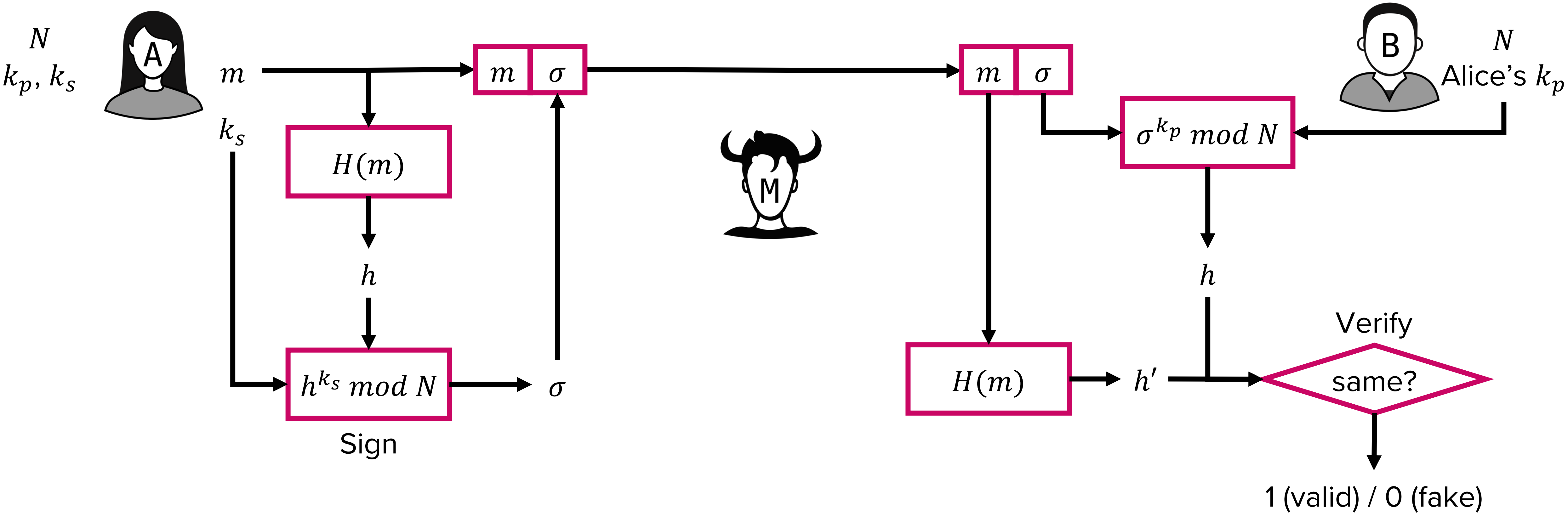
$m = m'$ holds. Return true

# Secure DS: Sign $H(m)$

- A better scheme: Hash the message first
  - Given: Message $m$, secret key $k_s$, public $k_p$, public parameter $N$
  - $h \leftarrow H(m)$
  - Sign $S(k_s, h)$: $\sigma \leftarrow h^{k_s} \ mod \ N$
  - Send $m$ and $\sigma$
  - Verify $V(k_p, \sigma, m)$:
    - $h \leftarrow H(m)$ // compute the hash of the received message $m$
    - $h' \leftarrow \sigma^{k_p} \ mod \ N$ // hash retrieved by decrypting $\sigma$
    - if $h = h'$ then return 1, else return 0

→ The previous forgery using $(m = 1, \sigma = 1)$ no longer works

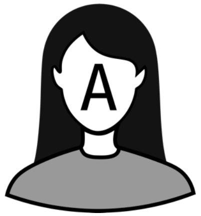
# Summary: Digital signature using hash and RSA




We can now provide integrity using an asymmetric scheme!

# Digital signature in practice

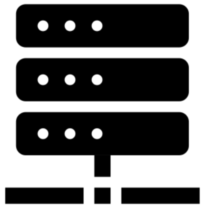
- Passwordless authentication of SSH (secure shell)




**Alice**

Initial setup for account “Alice”

1. Alice logs in using password
2. Alice registers her public key in `/home/Alice/.ssh/authorized_keys`
3. She can then disable password login in the SSH configuration file



**Server**

Passwordless login

1. Alice attempts to log in
2. she signs her identity with her secret key and sends it to the server
3. The server verifies the signature using the stored public key of Alice
4. If valid, Alice is granted access (no password required)

Only Alice can securely log in as long as her secret key is not leaked

# Rethinking the “authentication” problem

- Pizza prank scenario
  - Mallory creates an e-mail order:
  - Mallory signs the order with his own secret key
  - Mallory sends the signed order to the Pizza Store
  - The store responds, “Hey **Bob**, please send us your public key”
  - Mallory sends his public key
  - The store verifies the signature using Mallory’s public key, incorrectly assuming it is Bob’s
  - The store delivers four pepperoni pizzas to Bob, who is vegan

```
Dear Pizza Store,
I’d like to order four pepperoni pizzas.
Thank you,
- Bob
```

# Rethinking the "authentication" problem

- Pizza prank scenario
  - Mallory creates an e-mail order:
  - Mallory signs the order with

```
Dear Pizza Store,
I'd like to order four pepperoni pizzas.
Thank you,
- Bob
```

**Key question: Are public keys alone enough for strong authentication?**

  - The store verifies the signature using Mallory's public key, incorrectly assuming it is Bob's
  - The store delivers four pepperoni pizzas to Bob, who is vegan

# Cryptography roadmap

| Goal \ Scheme | Symmetric Key | Asymmetric Key |
|---|---|---|
| **Confidentiality** | ✅ One Time Pad (OTP)<br>✅ Block ciphers (DES, AES)<br>✅ Stream ciphers | ✅ DH secure key exchange<br>✅ ElGamal encryption<br>✅ RSA encryption |
| **Integrity & Authentication** | ✅ Message Authentication Code (MAC) | • Digital signature |
| **CIA at the same time** | ✅ Authenticated encryption | |

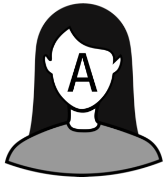
Really?

**Additional Tool**

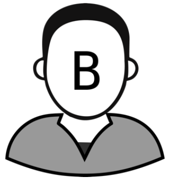
✅ Hash functions

# Certification Authorities

# Problem: Distributing public keys








**Alice:** Hey Bob, I want to talk to you. Give me your public key.

**Bob:** Generate secret-public key pair $< k_p^B, k_s^B >$

**Mallory:** Generate secret-public key pair $< k_p^M, k_s^M >$

**Bob:** Send $k_p^B$

**Mallory:** Store $k_p^B$

**Mallory:** Send $k_p^M$

**Alice:** Receive $k_p^M$ (Thinks it's Bob's public key)

**Alice:** Encrypt $m$ using $k_p^M$

**Alice:** Send $c = m^{k_p^M} \ mod \ N$

**Mallory:** Decrypt $c$ using $k_s^M$

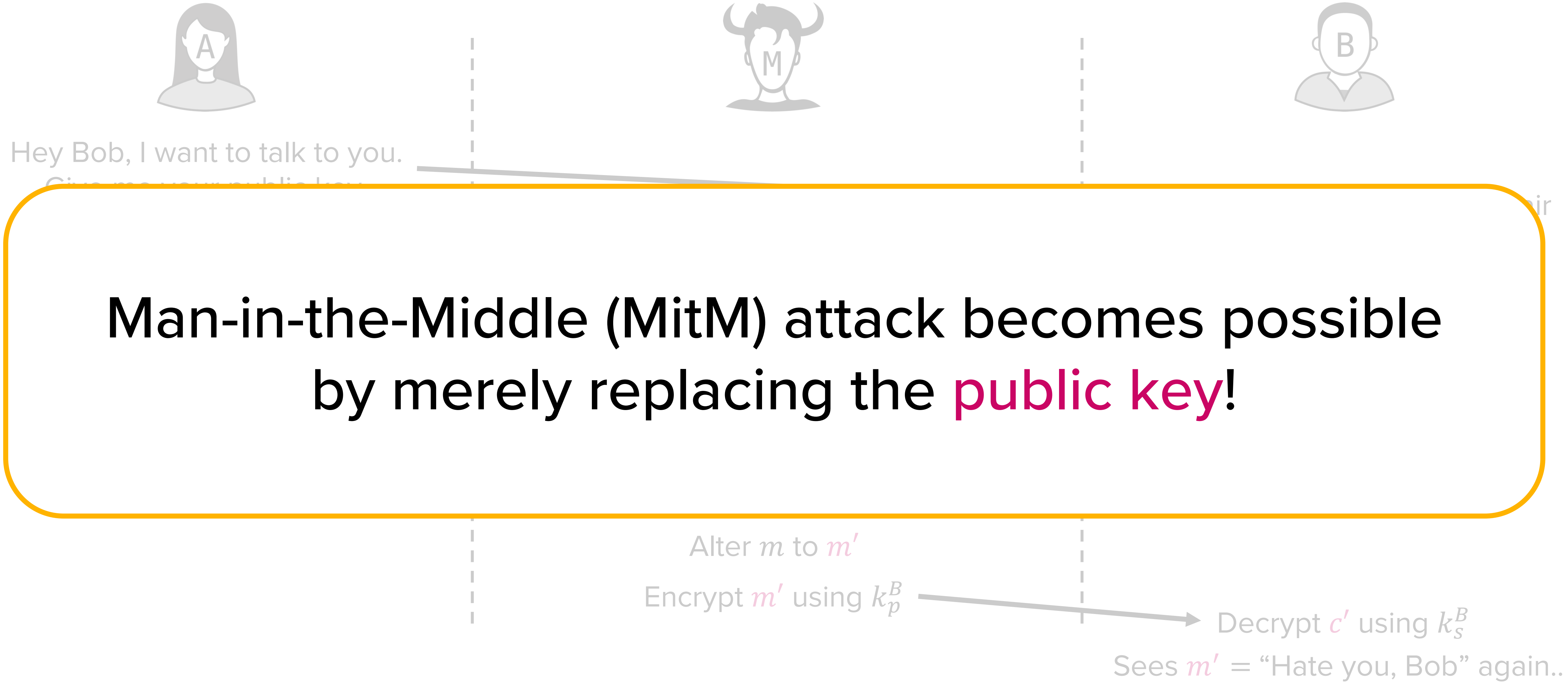
**Mallory:** Alter $m$ to $m'$

**Mallory:** Encrypt $m'$ using $k_p^B$

**Bob:** Decrypt $c'$ using $k_s^B$

**Bob:** Sees $m'$ = "Hate you, Bob" again..

# Problem: Distributing public keys




**Man-in-the-Middle (MitM) attack becomes possible by merely replacing the public key!**

# Problem: Distributing public keys

- Countermeasure idea
  - Sign Bob's public key to prevent tampering?
- Dilemma:
  - For verification, we require his **public key**
  - Yet, the purpose was to verify Bob's **public key** in the first place
  - Creates a circular problem!
    - Alice cannot fully trust any public key

We need a "root of trust"!

# Establishing root of trust: Trust-on-first-use (TOFU)

- Trust the public key that is used for the initial communication and warn the user if the key changes in the future
  - Rationale: Attacks are not frequent, so assume that the initial communication was not attacked
  - Used by SSH (Secure Shell)
    - Connect to a new server from my machine
    - Server's identification is saved on my machine (in `~/.ssh/known_hosts`)
    - If the server sends a different identification, we can suspect an MitM attack

```
> ssh root@65.109.131.51
@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@
@    WARNING: REMOTE HOST IDENTIFICATION HAS CHANGED!     @
@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@
IT IS POSSIBLE THAT SOMEONE IS DOING SOMETHING NASTY!
Someone could be eavesdropping on you right now (man-in-the-middle attack)!
It is also possible that a host key has just been changed.
The fingerprint for the ED25519 key sent by the remote host is
SHA256:qT2ctfBupHj1kOyC2WcJJPq76ge58iyiBf9sPDQTkdE.
Please contact your system administrator.
Add correct host key in /Users/ask/.ssh/known_hosts to get rid of this messag
e.
Offending ECDSA key in /Users/ask/.ssh/known_hosts:12
Host key for 65.109.131.51 has changed and you have requested strict checking
.
Host key verification failed.
```

Problem: Assumption is too strong

# Establishing root of trust: Certification Authority

- Certification Authority (CA) binds a public key to a specific entity (E)
  - Serves as a trusted third party (TTP)
- Procedure
  - Bob registers his public key with CA, providing a “proof of identity”
  - CA creates an identity binding (⋈) of Bob and his public key
  - The binding, digitally signed by CA’s secret key, serves as the certificate of Bob ($cert^B$)

$$id^B \leftarrow B \bowtie k_p^B \longrightarrow S(k_s^{CA}, id^B) \longrightarrow cert^B$$

# Certification Authority (CA)

- Now when Alice wants Bob's public key
  - Alice receives Bob's certificate ($cert^B$) from the CA
  - Alice applies the CA's public key to verify Bob's identity

$$cert^B \rightarrow V(k_p^{CA}, cert^B) \rightarrow id^B \leftarrow B \bowtie k_p^B$$

  - If Alice trusts the CA (root of trust), Alice can trust that Bob's public key is truly Bob's

# Building a practical CA

- Naïve idea: Make a central, trusted directory (TD) from which you can fetch anyone's public key
  - The TD has a public/secret key pair: $k_p^{TD}$ and $k_s^{TD}$
  - The directory publishes $k_p^{TD}$ to everyone
  - When someone requests Bob's public key, the directory sends a certificate for Bob's identity
    - $cert^B$, which is $B \bowtie k_p^B$ signed using $k_s^{TD}$
  - If you trust the TD, you trust every public key

# Building a practical CA

POSTECH

- Naïve idea: Make a central, trusted directory (TD) from which you can fetch anyone's public key
- Problems
  - Scalability: One directory will not have enough computing power to serve all entities in the entire world
  - Single point of failure:
    - If the TD fails, every service depending on TD becomes unavailable
    - If the TD is compromised, you cannot trust anyone
    - If the TD is compromised, it is extremely difficult to recover

# Building a practical CA

- Practical idea #1: Hierarchical trust model
  - The roots of trust may **delegate** the identity bindings and signing power to other authorities
    - Alice's public key is $k_p^A$ and I trust her to sign for POSTECH
    - Bob's public key is $k_p^B$ and I trust him to sign for the CSE department
    - Charlie's public key is $k_p^C$. I don't let him sign for anyone else
  - Hierarchy
    - Root CA
    - Alice and Bob are intermediate CAs

Solves the scalability problem

# Building a practical CA

- Practical idea #2: Multiple trust anchors
  - There are more than 200 root CAs in the world
  - Most operating systems provide a built-in list of trusted root CAs
  - Most web browsers, too

File /System/Library/Security/Certificates.bundle/Contents/Resources/TrustStore.html

Trust store version: 2024040500

**Trusted Certificates**

| Certificate name | Issued by | Type | Key size | Sig alg | Serial number | Expires | EV policy | Fingerprint (SHA-256) |
|---|---|---|---|---|---|---|---|---|
| AAA Certificate Services | AAA Certificate Services | RSA | 2048 bits | SHA-1 | 01 | 23:59:59 Dec 31, 2028 | Not EV | D7 A7 A0 FB 5D 7E 27 31 D7 71 E9 48 4E BC DE F7 1D 5F 0C 3E 0A 29 48 78 2B C8 3E E0 EA 69 9E F4 |
| AC RAIZ FNMT-RCM | AC RAIZ FNMT-RCM | RSA | 4096 bits | SHA-256 | 5D 93 8D 30 67 36 C8 06 1D 1A C7 54 84 69 07 | 00:00:00 Jan 1, 2030 | Not EV | EB C5 57 0C 29 01 8C 4D 67 B1 AA 12 7B AF 12 F7 03 B4 61 1E BC 17 B7 DA B5 57 38 94 17 9B 93 FA |
| ACCVRAIZ1 | ACCVRAIZ1 | RSA | 4096 bits | SHA-1 | 5E C3 B7 A6 43 7F A4 E0 | 09:37:37 Dec 31, 2030 | Not EV | 9A 6E C0 12 E1 A7 DA 9D BE 34 19 4D 47 8A D7 C0 DB 18 22 FB 07 1D F1 29 81 49 6E D1 04 38 41 13 |

Solves the single-point-of-failure problem

# Building a practical CA

- New problem: Revocation
  - What if a CA messes up and issues a bad certificate?
    - e.g., CA: "Bob's public key is $k_p^M$"
  - Everyone will trust the wrong public key
  - If Mallory signs messages, people will think Bob did

We need to be able to revoke bad certificates!

# Building a practical CA – Revocation

- Approach #1: Each certificate has an expiration date
  - When the certificate expires, request a new certificate from a CA
  - Bad certificates will eventually become invalid once they expire
- Strength:
  - No bad certificate remain forever
- Weakness: Everybody must renew frequently (overhead)
  - Frequent renewal: More security, less usability
  - Infrequent renewal: Less security, more usability

# Building a practical CA – Revocation

- Approach #2: Periodically release a list of invalidated certificates
  - Users periodically download a Certification Revocation List (CRL)
- Strength:
  - Invalid certificates are revoked as soon as a user downloads the CRL
- Weakness:
  - Size of the list grows linearly to the number of revoked certificates
  - Cannot know which certificates are revoked before downloading CRL

# Current certificate standard: X.509

- Certificate contains
  - Issuer’s name
  - Entity’s name, address, domain name, …
  - Entity’s public key
  - Digital signature of the certificate (signed with the issuer’s secret key)

- Core components
  - Certificates and CAs
  - Certificate revocation list

# Summary

- Certificate: A signed attestation of identity
- Trusted directory: One server holds all keys
- Certificate authorities: Provide delegated trust from a pool of multiple root CAs
  - Root CA can sign certificates for intermediate CAs
  - Certificates can be revoked (timed expiry or revocation list)

# Cryptography roadmap

| Goal \ Scheme | Symmetric Key | Asymmetric Key |
|---|---|---|
| **Confidentiality** | ✅ One Time Pad (OTP)<br>✅ Block ciphers (DES, AES)<br>✅ Stream ciphers | ✅ DH secure key exchange<br>✅ ElGamal encryption<br>✅ RSA encryption |
| **Integrity & Authentication** | ✅ Message Authentication Code (MAC) | ✅ Digital signature + CA |
| **CIA at the same time** | ✅ Authenticated encryption | • ??? |

**Additional Tool**

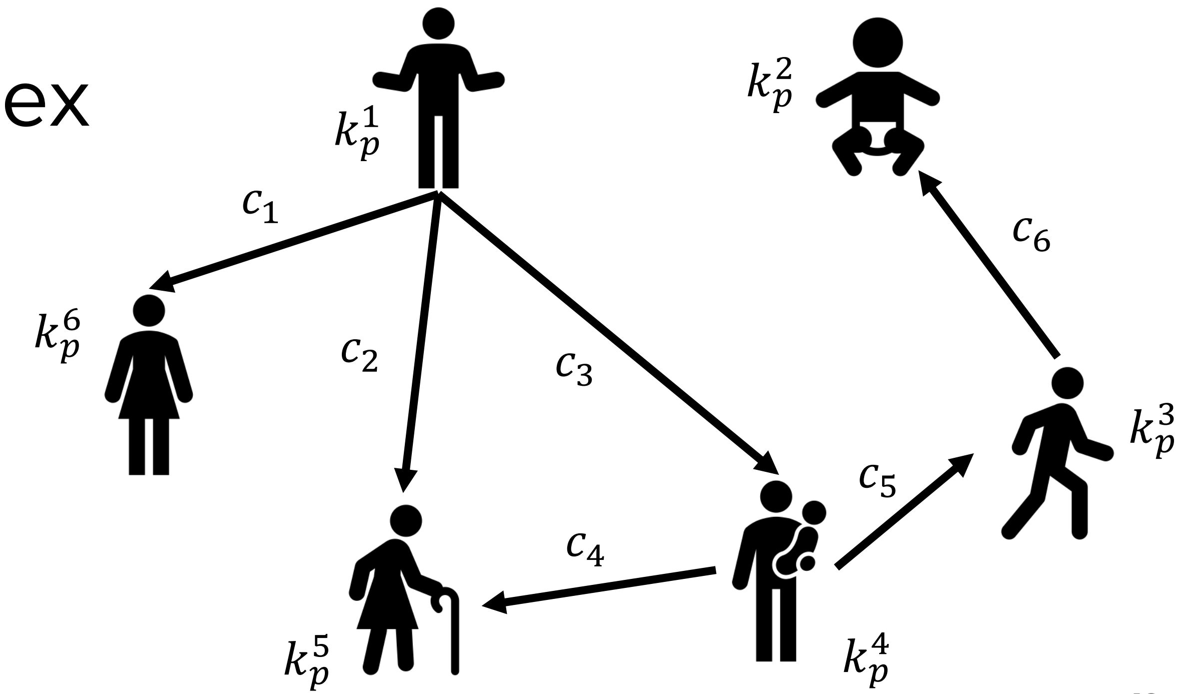
✅ Hash functions

# Multi-user Setting and Signcryption

# Multi-user setting complications

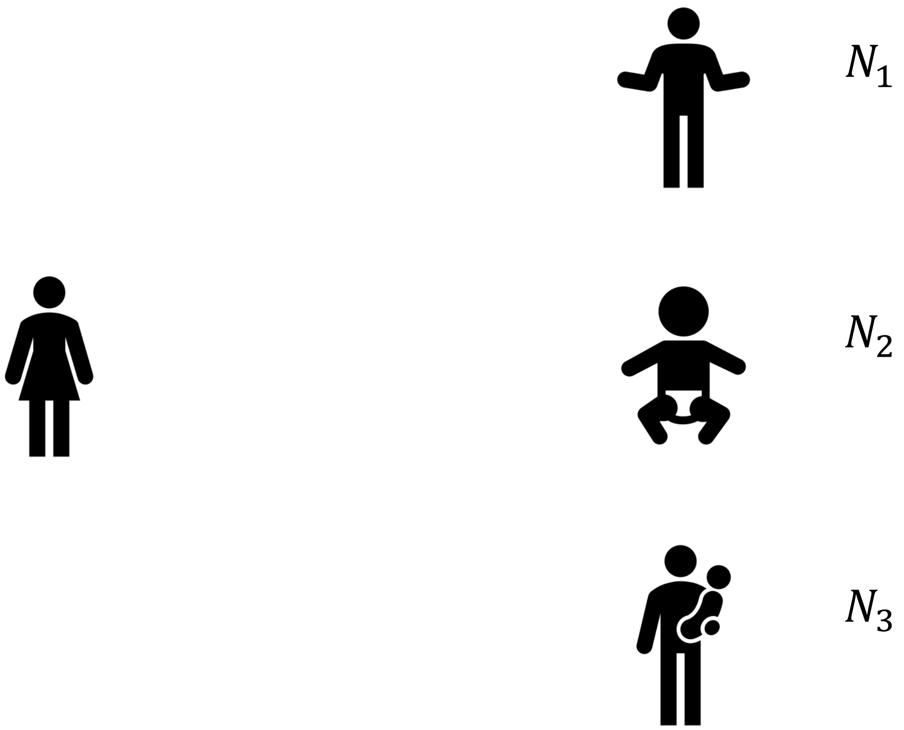
- Security of asymmetric schemes considered a single user
  - "Can a sender have confidentiality?"
  - "Can a receiver verify a signature?"

- Real world is much more complex



# Multi-user setting complications

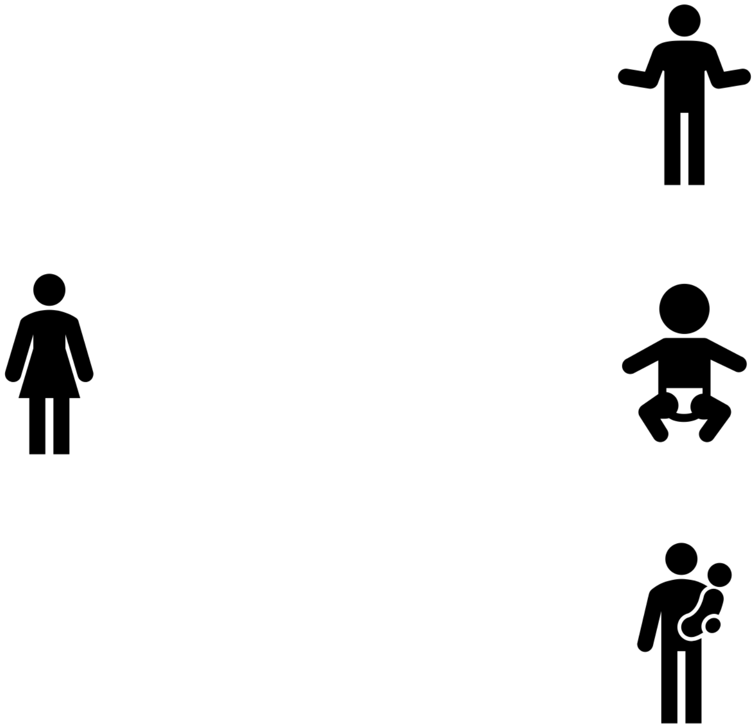
- Known real-world attack: Hastad-type attack on RSA




Three people select different large numbers $N_1$, $N_2$, $N_3$ for RSA key generation

# Multi-user setting complications

- Known real-world attack: Hastad-type attack on RSA



$N_1$
$k_p^1 = 3$

$N_2$
$k_p^2 = 3$

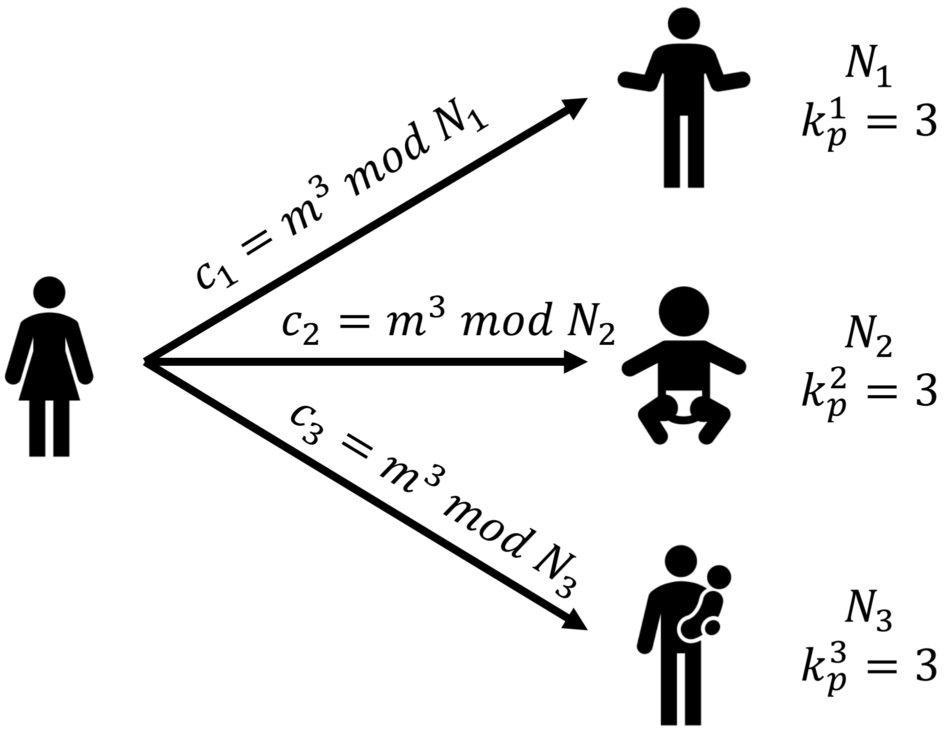
$N_3$
$k_p^3 = 3$

Three people happen to select
the same public key $k_p^i$
that are coprime to the totient of each N
(Recall, if $N = pq$,
its totient $T = (p-1)(q-1)$)

e.g., $k_p^1 = k_p^2 = k_p^3 = 3$

# Multi-user setting complications

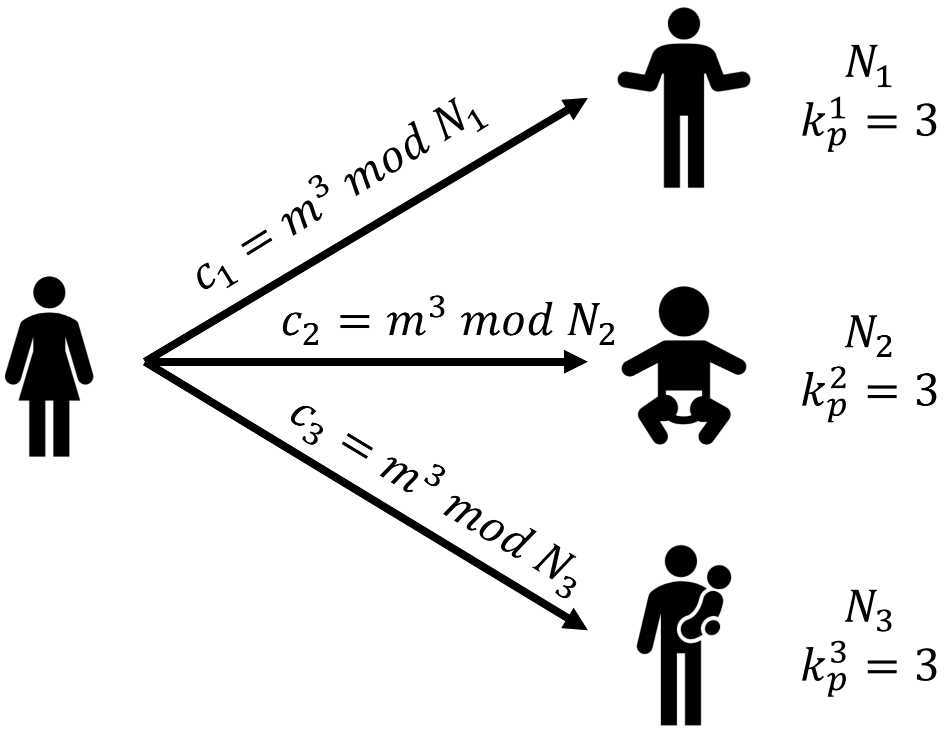
- Known real-world attack: Hastad-type attack on RSA




The sender wants to send $m$ and RSA-encrypts it using $N_i, k_p^i$ for each recipient

# Multi-user setting complications

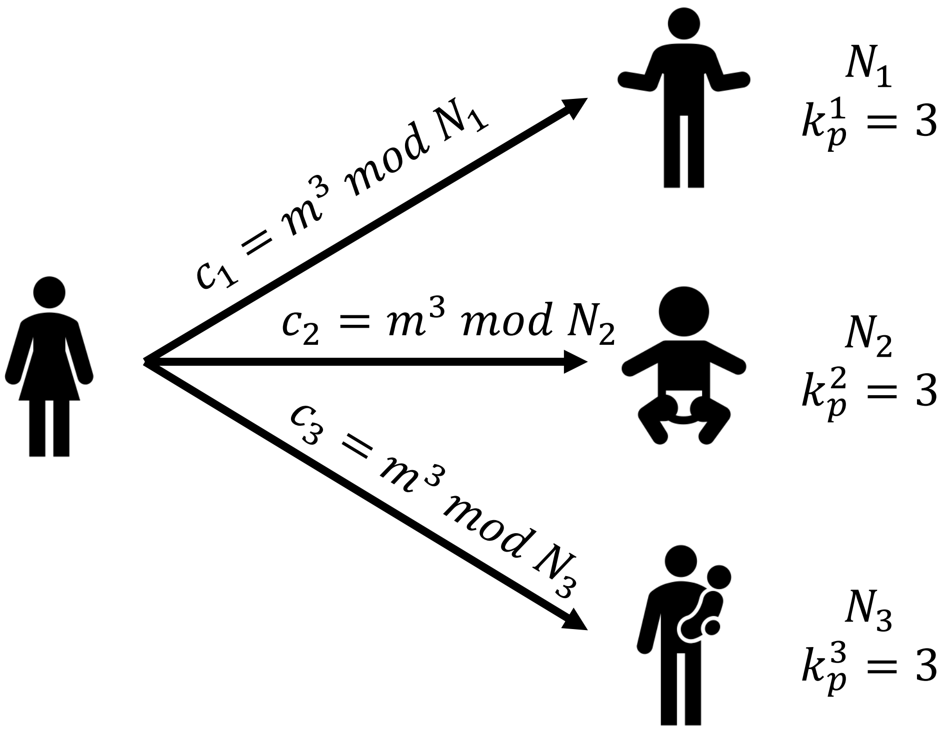
- Known real-world attack: Hastad-type attack on RSA




Only the three recipients, individually, should be able to decrypt $m$ from $c_i$ using their $k_s^i$

# Multi-user setting complications

- Known real-world attack: Hastad-type attack on RSA




If $N_1, N_2, N_3$ are relatively prime,
then by Chinese Remainder Theorem,

- $c_1 = m^3 \ mod \ N_1$
- $c_2 = m^3 \ mod \ N_2$
- $c_3 = m^3 \ mod \ N_3$

can be combined to find:

$$c = m^3 \ mod \ N_1 N_2 N_3$$

Since $m^3 < \mathrm{N_1 N_2 N_3}$, we get $m = \sqrt[3]{c}$

$m$ can be completely recovered using public keys

# Signcryption

- Signcryption is a public key-based primitive that assures confidentiality, integrity, and authenticity at the same time
  - Not by separately utilizing encryption and digital signatures
  - Goal is to combine encryption and signing into a single operation
- e.g., sign-then-encrypt?
  - Signing involves an encryption (using a secret key)
  - Encrypting involves another encryption (using a public key)

  → Redundancy (== inefficiency)

# Signcryption

- Signcryption presents significant challenges:
  - Strong security should be provided:
    - Indistinguishability under chosen plaintext/ciphertext attacks
    - Unforgeability
  - Multi-user setting poses more challenges
    - e.g., Hastad-type attack

- As of now, no provably-secure algorithm has been developed

# Cryptography roadmap

| Goal \ Scheme | Symmetric Key | Asymmetric Key |
|---|---|---|
| **Confidentiality** | ✅ One Time Pad (OTP)<br>✅ Block ciphers (DES, AES)<br>✅ Stream ciphers | ✅ DH secure key exchange<br>✅ ElGamal encryption<br>✅ RSA encryption |
| **Integrity & Authentication** | ✅ Message Authentication Code (MAC) | ✅ Digital signature + CA |
| **CIA at the same time** | ✅ Authenticated encryption | None |

**Additional Tool**

✅ Hash functions

# Coming up next

- What do we do in the real world?
  - Applications (e.g., Internet Security Protocols)
  - Incidents of crypto-based attacks

# Questions?